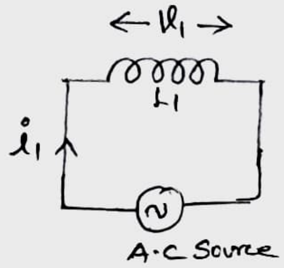
STATICALLY AND DYNAMICALLY INDUCED EMF

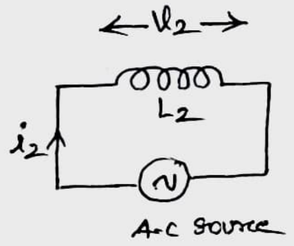
no rotation

electromotive force.

STATICALLY INDUCED EMF:

- * The coil remains stationary with respect to flux, but the flux throught it changes with time. The emf induced is known as statically induced emf.





$$V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \; ; \; V_2 = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

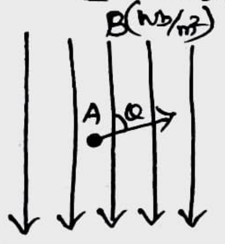
Statically Induced emf(EMF) can be further classified as:

(i) Self Induced emf ⇒ emf induced due

(ii) Mutual Induced emf ⇒ emf induced due to current in the neiboring coil.

DYNAMICALLY INDUCED EMF:

- * Flux density (B) distribution remains constant and stationary but the coil moves relatively to it. The emf induced is known as dynamically induced (or motional) emf. Example: Machine (all rotating machine)



Let:

A → be the conductor.

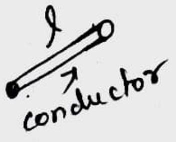
B → Magnetic flux density in Wb/m².

l → length of the conductor.

V → velocity of conductor in m/sec.

α → Angle between direction of magnetic flux and direction of velocity of conductor in degrees.

e → is voltage produced in the conductor



Note: The conductor moves at an angle α with the direction of the lines of force the induced emf is $e = BlV \sin\alpha$ (volts) This is called dynamically induced emf.

FUNDAMENTALS OF MAGNETIC CIRCUITS

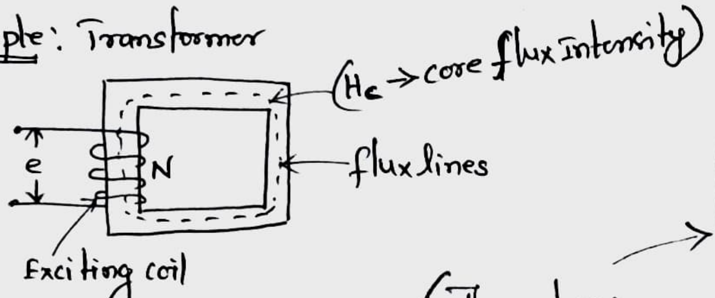
MAGNETIC CIRCUIT: (The complete closed path followed by the lines of flux is called a magnetic circuits. Eg: Magnetic devices such as Toroids, Transformers, motors, generators, and relays, consider magnetic circuit.)

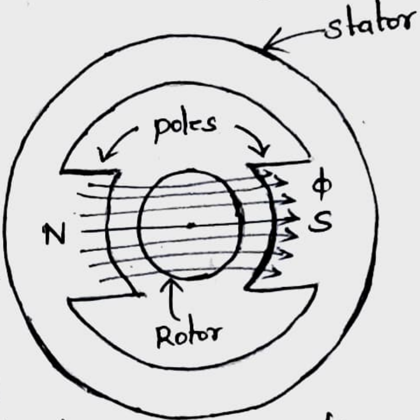
- * In electrical machines, the magnetic circuits may be formed by ferromagnetic materials only (as in Transformer) or by ferromagnetic materials in conjection with an airmedium (as in rotating machines).

- * The complete closed path followed by the lines of flux is called a magnetic circuit.

- * Magnetic devices such as toroids, transformers, motors, generators, and relays may be considered as magnetic circuits.

Example: Transformer

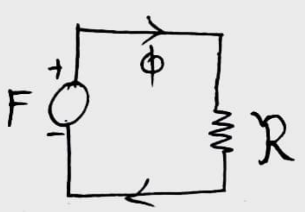




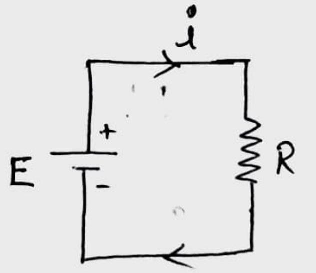
(The rotor is physically isolated from the stator by the air gap)

Example: Rotating machine. (or) Electrical machine.

Analogy between Magnetic circuit and Electric circuit:



Magnetic equivalent circuit



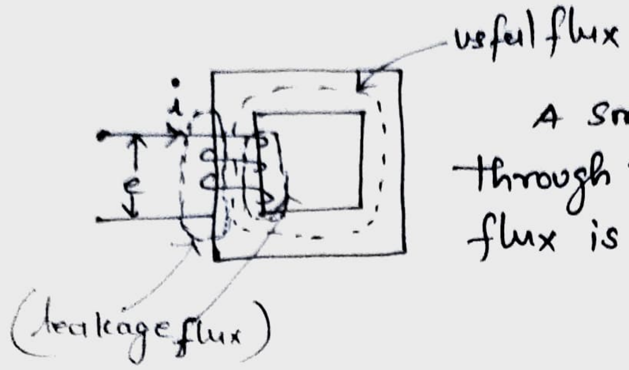
Electric equivalent circuit.

σ → conductivity of materials
μ → permeability
A → Area of Cross Section of the conductor
l → length of (or) core length

Table: Electrical versus Magnetic circuits

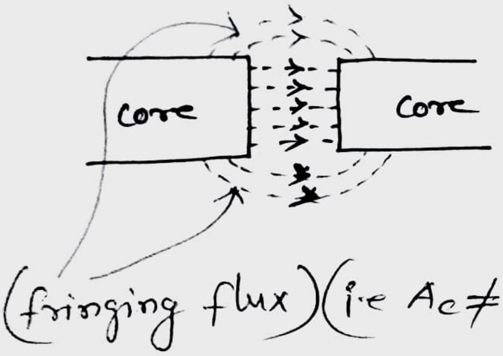
	Electric Circuit	Magnetic circuit
Driving Force	EMf (E) volts	Mmf (F) = IN (ATs)
produces	Current ($i = E/R$) Amps	Flux ($\phi = F/\mathcal{R}$) Wb
Limited by	Resistance $R = \frac{l}{\sigma A}$	Reluctance ($\mathcal{R} = l/\mu A$) AT/Wb

LEAKAGE FLUX:



A small amount of flux always leaks through the surrounding air. This stray flux is called the leakage flux.

FRINGING FLUX:



(i.e $A_c \neq A_g$)

At an air-gap in a magnetic core, the flux fringes out into neighbouring air path is known as fringing flux.

OHM'S LAW for MAGNETIC CIRCUIT.

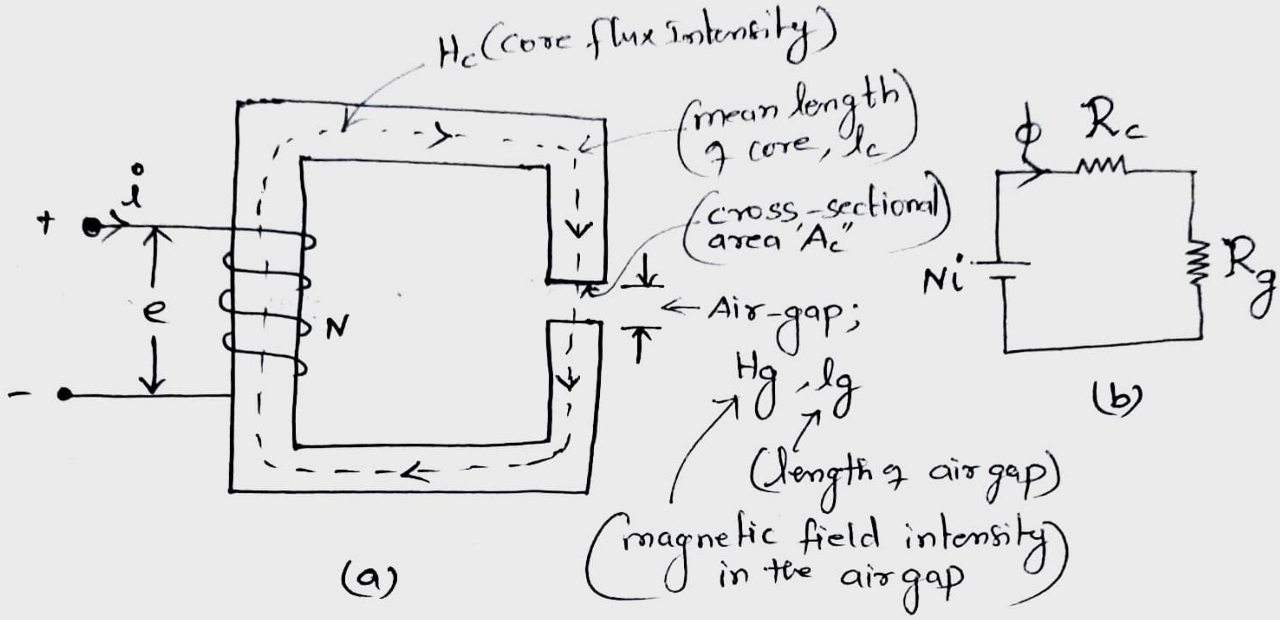
According to ohm's law the flux ϕ is directly propotional to the force (Ni) and inversly propotional to the Reluctance "R".

$$\text{Flux}, \ \phi = \frac{\mathcal{F}}{R} \quad \text{webers.}$$

clearly define the MMF and EMF.

Since 'N' is the number of coil turns and i the exciting current in amperes, the product $\mathcal{F} = Ni$ has the units of ampere-turns (AT) and is the cause of establishment of the magnetic field. It is known as the magnetomotive force in analogy to the electromotive force (emf) which established current in an electric circuit $V = iR$ or $i = \frac{V}{R}$.

A TYPICAL MAGNETIC CIRCUIT CORE WITH AN AIR-GAP



composite structure (a) Magnetic circuit with airgap.
(b) Magnetic equivalent circuit.

⇒ It is assumed that the air-gap is narrow and the flux comming out of the core passes slight down the air-gap such that the flux density in the air-gap is the same as in the core.

⇒ Actually as the flux in the gap fringes out so that the gap flux density is somewhat less than that of the core.

⇒ Further, let the core permeability 'μ_c' be regarded as constant (linear magnetization characteristic)

Let us consider the simple composite structure.

→ The driving force in the magnetic circuit is the

$$\text{mmf}, \ F = Ni = \underbrace{\boxed{H_c l_c}}_{F_c} + H_g l_g$$

The quantiti Ni is called the magnetomotive force (mmf) F and it's unit is ampear-turn.

→ core medium Reluctance (or) Core Reluctance $R_c = \frac{l_c}{\mu_c A_c}$

core permeability (μ_c) — Area of cross section of core (A_c)

→ Air-gap medium Reluctance (or) Air-gap Reluctance $R_g = \frac{l_g}{\mu_0 A_g}$

$$\therefore \phi = \frac{Ni}{R_c + R_g}$$

$$\phi = \frac{Ni}{\left(\frac{l}{\mu A}\right)} = BA = \mu H A = \mu \frac{Ni}{l_e} A = \frac{Ni}{\boxed{\frac{l}{\mu A}}}$$ ← Reluc

$$\phi = \frac{Ni}{R}$$ ← Reluctance

$$Ni = H_c l_c + H_g l_g \longrightarrow ①$$

where:

l_c → is the mean length of the core.

l_g → is the length of the air gap.

note: $B = \mu H$

$H = \frac{B}{\mu}$

so

$H_c = \frac{B_c}{\mu_c}$

$$\therefore Ni = \overset{H}{\frac{B_c}{\mu_c}} l_c + \frac{B_g}{\mu_0} l_g \longrightarrow ②$$

Assuming that all the core flux passing stright down the air-gap (it means no fringing)

$$\therefore B_g = B_c$$

Cross sectional area of core — Cross sectional area of Air gap.

$$\phi = B_c A_c = B_g A_g \longrightarrow ③$$

∴ sub eqn ③ in eqn ②

$$Ni = \frac{B_c}{\mu_c} l_c + \frac{B_g}{\mu_0} l_g$$

~~Note~~

$$Ni = \frac{\frac{\phi}{A_c}}{\mu_c} l_c + \frac{\frac{\phi}{A_g}}{\mu_0} l_g$$

$$Ni = \frac{\phi l_c}{\frac{\mu_c}{A_c}} + \frac{\phi l_g}{\frac{\mu_0}{A_g}}$$

$$Ni = \frac{\phi l_c}{\mu_c} \frac{1}{A_c} + \frac{\phi l_g}{\mu_0} \frac{1}{A_g}$$

$$Ni = \frac{\phi l_c}{\mu_c A_c} + \frac{\phi l_g}{\mu_0 A_g}$$

$$Ni = \phi \left(\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_g} \right)$$

$$F = \phi(R_c + R_g) = \phi R_{eq} \longrightarrow ④$$

$$\phi = \frac{F}{R_c + R_g} = \frac{F/R_g}{1 + \frac{R_c}{R_g}}$$

But $R_c = \frac{l_c}{A_c \mu_c}$, $R_g = \frac{l_g}{A_g \mu_0}$

$$\frac{R_c}{R_g} = \frac{\frac{l_c}{A_c \mu_c}}{\frac{l_g}{A_g \mu_0}}$$

$$\frac{R_c}{R_g} = \frac{l_c}{\cancel{A_c} \mu_c} \times \frac{\cancel{A_g} \mu_0}{l_g}$$

$$\frac{R_c}{R_g} = \frac{l_c}{\mu_c} \times \frac{\mu_0}{l_g}$$

$$\frac{R_c}{R_g} = \frac{\mu_0 l_c}{\mu_c l_g} << 1$$

because $\mu_0 = 4\pi \times 10^{-7}$

$\mu_c = \mu_r \mu_0$

$= 2000 \times 4\pi \times 10^{-7}$

because μ_c is 2000 to 6000 times μ_0 in ferromagnetic materials.

$$\phi = \frac{F/R_g}{1 + \frac{R_c}{R_g}}$$

$$\phi = \frac{F/\cancel{R_g}}{\frac{R_g + R_c}{\cancel{R_g}}}$$

$$\phi = \frac{F}{R_g + R_c}$$

note: $A_c = A_g$
it means no fringing

PRINCIPLE OF ELECTROMECHANICAL ENERGY CONVERSION

⇒ Electromechanical energy conversion takes place via the medium of a magnetic or electric field - the magnetic field being most suited for practical conversion devices. Because of the inertia associated with mechanical moving members, the fields must necessarily be slowly varying i.e. quasistatic in nature

⇒ The conversion process is basically a reversible one through practical devices may be designed and constructed to particularly suit one mode of conversion or the other.

⇒ So we have to need the understanding of the principle of electromechanical energy conversion.

THREE BASIC PRINCIPLE OF ELECTROMECHANICAL ENERGY CONVERSION

⇒ The three basic principles are Induction, Intraction and alignment:

Induction: pertains to the emf induced in a coil when the coil links changing flux linkages.

Interaction: pertains to the development of force or torque when fields produced by stator as well as rotor intract with each other.

Alignment: pertains to the development of reluctance force or torque. This torque is present when the reluctance seen by the working flux changes with rotor movement.

Based on the principle of conversion of energy, write an energy balance equation for a motor.

⇒ According to the principle of conversion of energy, energy can neither be created nor distroyed, it can merely be converted from one form into another.

The energy balance equation for a motor.

$$\begin{bmatrix}\text{Total electrical} \\ \text{energy input}\end{bmatrix} = \begin{bmatrix}\text{Mechanical} \\ \text{Energy output}\end{bmatrix} + \begin{bmatrix}\text{Total Energy} \\ \text{Stored}\end{bmatrix} + \begin{bmatrix}\text{Total Energy} \\ \text{Dissipated}\end{bmatrix}$$

ROLE OF MAGNETIC FIELD IN ELECTROMAGNETIC ENERGY CONVERSION:

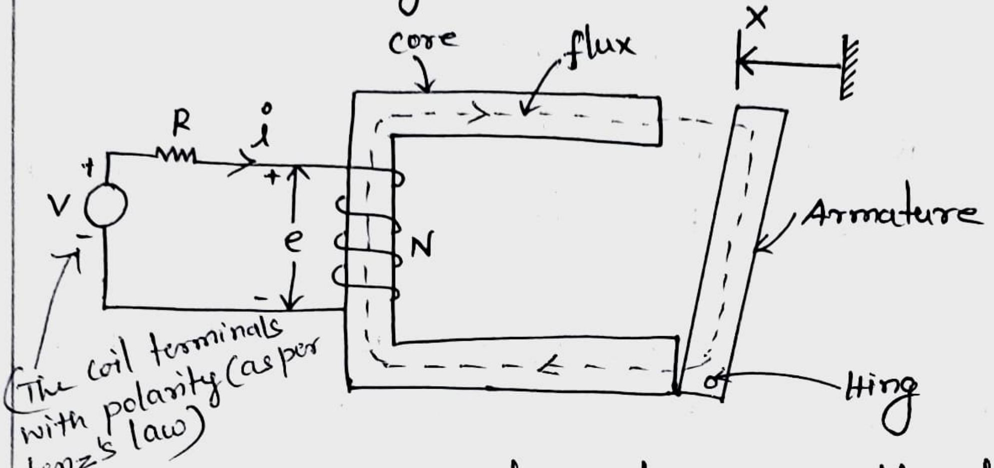
⇒ Torque or Force - producing devices with limited mechanical motion.

Example: of such devices are electromagnets, relays, moving-iron instruments, moving-coil instruments, actuators etc.

SINALE EXCITED AND MULTI EXCITED MAGINETIC FIELD SYSTEM

SINGLE EXCITED MAGNETIC FIELD SYSTEM:

Example: Reluctance motor, single phase Transformer and Relay coil.



Eg: The magnetic system of an attracted armature relay.

⇒ The coil as an ideal loss-lees coil.

⇒ The coil current causes (produces) magnetic flux to be established in the magnetic circuit.

⇒ Assume all the flux ϕ is confined to the iron core and there-four links all the 'N' turns creating the coil flux linkages of

(note: The leakage flux does not take part in the energy conversion process)

$$\underset{\text{flux linkages}}{\lambda} = \overset{\text{No. of turns}}{N}\underset{\text{flux}}{\phi} \longrightarrow (1.a)$$

⇒ flux linkages causes a reaction of

$$e = \frac{d\lambda}{dt} \longrightarrow (2.a)$$

Note: $Ni = \mathcal{F}$ (mmf) ↓ magnetomotive force

$d\lambda$ → Instantaneous flux linkages

⇒ The associated circuit equation is

$$v = iR + e$$

$$v = iR + \frac{d\lambda}{dt} \longrightarrow (3.a)$$

(multiply both sides of eqn 3.a by $i dt$)

$$vi\,dt = i\,dt\left(iR + \frac{d\lambda}{dt}\right)$$

$$vi\,dt = i^2R\,dt + \frac{i\,dt\,d\lambda}{dt}$$

$$vi\,dt = i^2R\,dt + i\,d\lambda$$

$$vi\,dt - i^2R\,dt = i\,d\lambda$$

$$vi\,dt - ii\,R\,dt = i\,d\lambda$$

$$(v - iR)\,i\,dt = i\,d\lambda$$

$$e = i\,dt = i\,d\lambda$$

Note: $e = v - iR$

⇒ The electrical energy I/p into the ideal coil due to the flow of current i in time dt is

$$dW_e = ei\,dt \longrightarrow \text{④·a}$$

Note: $dW_e = ei\,dt = i\,d\lambda$
i.e. $= \frac{d\lambda}{dt} i\,dt = i\,d\lambda$

⇒ Assuming for the time being that the armature is held fixed at position x, all the I/p energy is stored in the magnetic field. Thus.

$$dW_e = ei\,dt = dW_f \longrightarrow \text{⑤·a}$$

dW_f → is the change in field energy in time dt.

eqn ②·a is substituted in eqn ⑤·a

$$dW_e = \frac{d\lambda}{dt} i\,dt = dW_f$$

$$dW_e = i\,d\lambda = \mathcal{F}\,d\phi = dW_f \longrightarrow \text{⑥·a}$$

Note: $\lambda = N\phi$

$dW_e = i\,d\lambda$
$dW_e = i\,dN\phi$
$dW_e = Ni\,d\phi$
$dW_e = \mathcal{F}\,d\phi$

The relation ship i–λ or $\mathcal{F}$–λ is a functional one corresponding to the magnetic circuit which in general is nonlinear (and is also h tory-dependent, i.e it exhibits hysteresis)

Note: The total mmf force can be considered as the product of No. of turns and current.

The energy obsorbed by the field for finite change in flux linkages for flux is obtained from eqn (6.a)

$$\Delta W_f = \int_{\lambda_1}^{\lambda_2} i(\lambda) d\lambda = \int_{\phi_1}^{\phi_2} \mathcal{F}(\phi) d\phi \longrightarrow (7.a)$$

The energy obsoorbed by the magnetic system to establish flux ϕ (or flux linkages λ) from initial zero flux is.

$$W_f = \int_0^{\lambda} i(\lambda) d\lambda = \int_0^{\phi} \mathcal{F}(\phi) d\phi \longrightarrow (8.a)$$

The energy of the magnetic field with given mechanical configuration when its state corresponds to flux ϕ (or flux linkages λ)

$$i = i(\lambda, x)$$

if λ is the independent variable or as

$$\lambda = \lambda(i, x)$$

if i is the independent variable.

∴ the field energy eqn (8.a) is in general a function two variables.

$$W_f = W_f(\lambda, x) \longrightarrow (9.a)$$

or

$$W_f = W_f(i, x) \longrightarrow (10.a)$$

eqn (9.a) & (10.a) field energy is determined by the insteneous values of the system state ((λ, x) or (i, x)) and is independent the path followed by these states to reach the present value.

This means that the field energy at any instant is history independent.

A change in λ with fixed x causes electric - magnetic energy interchange governed by the circuit eqn (3.a) and the energy eqn (6.a)

Illy If 'x' is allowed to change with fixed λ, energy will interchange between the magnetic circuit and Mechanical system.

i-λ Relation ship:

Really

⇒ The i-λ relationship is indeed the magnetization curve which varies with the configuration variable x (eg: the air gap between the armature and core varies with - position 'x" of the armature.

⇒ The total reluctance of the magnetic path decreases as 'x' increases)

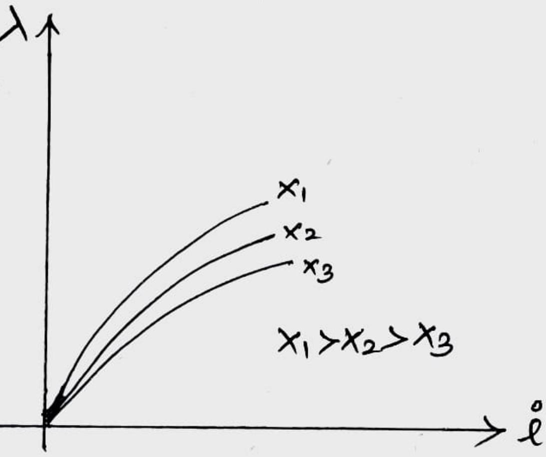
⇒ The i-λ relationship for various values of x is indicated in the graph.

Note: Relation between Current and flux linkages.

'x' ↑ in +ve direction the airgap reduced and hence reluctance ↓

This case the flux linkages for a small magnitude of current (it means saturation is neglected)

(i → is instantaneous current)



i-λ Relationship with variable 'x'

Co-Energy : (The differential variables of system related so current is termed as co-energy $W'_f = \int_0^{i} i(\lambda) di$.)

$$\int_{i=0}^{i} \lambda(i) di = W_f$$

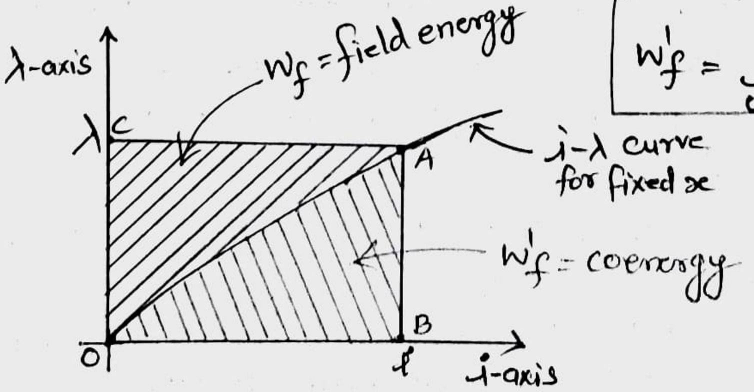
⇒ Co-Energy defined as

$$W'_f(i,x) = i\lambda - W_f(\lambda,x)$$

⇒ where in λ as λ(i,x), the independent variables of W'_f become i and x.

⇒ The area of OABO is complementary are of the i-λ rectangle.

$$\boxed{W'_f = \int_0^{\lambda} i\lambda \, di}$$



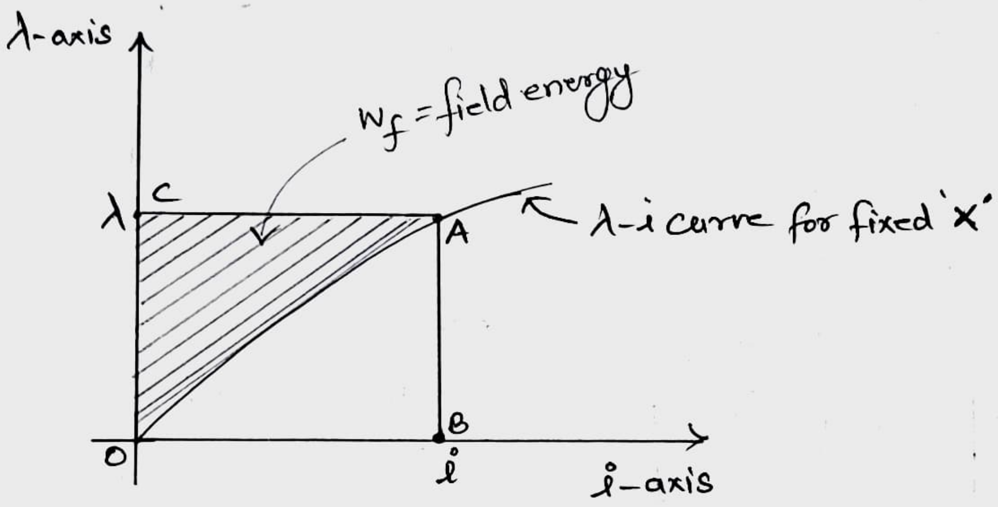
Q: what is the significance of co-energy.

$W'_f = \int_0^{i} \lambda di$ (or) $W'_f = \int_0^{\lambda} \lambda di$

FIELD ENERGY

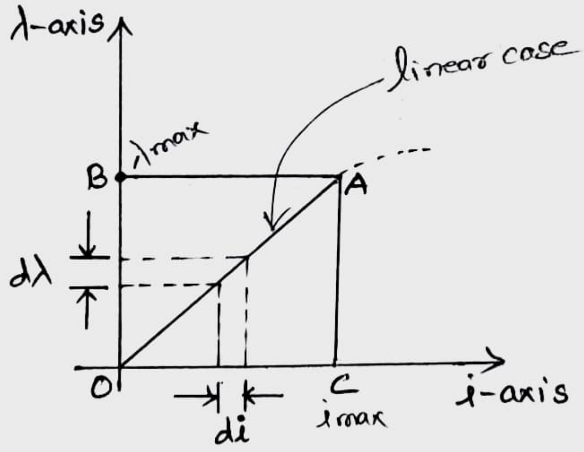
⇒ The field energy is the area "OACO" between the λ-axis and i-λ curve (as shown below). The energy absorbed by the magnetic system to establish flux ϕ (or flux linkage λ) from initial zero flux is.

$$\boxed{W_f = \int_0^{\lambda} i(\lambda)\,d\lambda = \int_0^{\phi} F(\phi)\,d\phi}$$



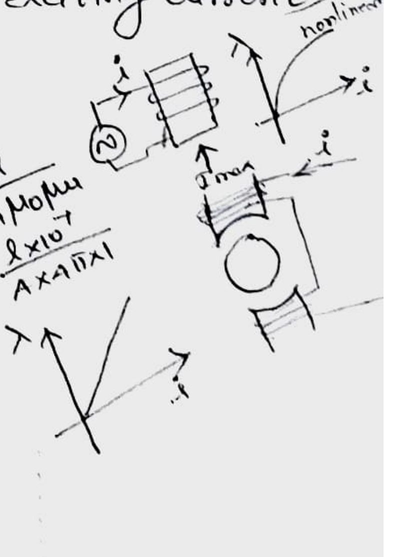
LINEAR CASE:

⇒ Electromechanical energy conversion devices are built with air-gap in the magnetic circuit which ~~seperate~~ serve seperate the stationary and moving members. As a result the i-λ relationship of the magnetic circuit is almost linear.



(dλ → Small incremental in λ
di → Small increment of Exciting current)

Airgap Reluctance $= \frac{l}{A\mu_0\mu_r} = \frac{l\times10^{7}}{A\times4\pi\times1}$



Area OABO = Area OACO

$W_f = W'_f$

(Upper area and lower area both are equal) $W_f + W'_f$ = Area OABO + OACO

$\boxed{W_f + W'_f} = \boxed{i\lambda}$ ← (area of Rectangle = l×b)

$2W_f = \boxed{i\lambda}$ ← actually imax, λmax

$$W_f = \frac{1}{2} i\lambda$$

$$W_f = \frac{1}{2} i N\phi$$

$$W_f = \frac{1}{2} \mathcal{F}\phi \rightarrow \text{(11.a)}$$

$$W_f = \frac{1}{2} R\phi^2 \rightarrow \text{(11.a)}$$

Note! $\lambda = N\phi$

$mmf = \mathcal{F} = Ni$

$$R = \mathcal{F}/\phi = \frac{mmf}{\phi}$$

where 'R' reluctance q the magnetic circuit.

——×——×——

$\lambda = N\phi$

$Ni = N\phi$

$$i = \frac{\cancel{N}\phi}{\cancel{N}}$$

$i = \phi$

The self inductance q the coil is defined as the magnetic flux linkages per Ampere.

$$L = \lambda/i$$

$$W_f = \frac{1}{2} i\lambda$$

The field energy can be expressed as

$$W_f = \frac{1}{2} \frac{\lambda}{L} \lambda$$

$$\boxed{W_f = \frac{1}{2} \frac{\lambda^2}{L}} \rightarrow \text{(12.a)}$$

$\Rightarrow \lambda = Li$

also $i = \lambda/L$

$$\Rightarrow W_f = \frac{1}{2} \frac{\lambda}{L} \cdot \lambda$$

$$W_f = \frac{1}{2} \frac{\lambda^2}{L}$$

$$W_f = \frac{1}{2} \frac{L^{\cancel{2}} i^2}{\cancel{L}}$$

$$W_f = \frac{1}{2} L i^2$$

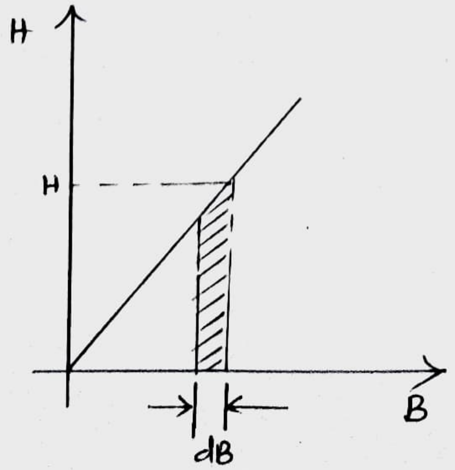
In the linear case the Inductance L is independent q i but is a function q configration x. thus the field energy is a special function q two independent variable λ and x.

Nφ, φ depend upon i. independent variable.

$$W_f(\lambda, x) = \frac{1}{2} \frac{\lambda^2}{L(x)} \longrightarrow \text{(13.a)}$$

'L' depends upon 'x'

ENERGY DENSITY!



note!

$$H = \frac{\mathcal{F}}{l} \Rightarrow$$ magnetic flux Intensity in AT/m

$$B = \frac{\phi}{A} \Rightarrow$$ magnetic flux density in wb/m² or T

$$W_f = \int_0^B H\cdot dB$$

i.e: $H = \frac{B}{\mu}$

Note:
$H = \frac{B}{\mu}$
$B = \mu H$
$\mu = \mu_0 \mu_r$

$$W_f = \int_0^B \frac{B}{\mu}\cdot dB$$

$$W_f = \frac{B}{\mu}\, B$$

$$W_f = \frac{B^2}{\mu}$$

$$W_f = \frac{1}{\mu}\, \frac{B^2}{2}$$

check,

? $\boxed{W_f = \frac{1}{2}\, \frac{B^2}{\mu}}$ J/m^3 $\longrightarrow$ (14.a)

The linear case that coenergy is numerically equal to energy.

i.e $W'_f = W_f = \frac{1}{2}\lambda i = \frac{1}{2} F\phi = \frac{1}{2} P F^2 \longrightarrow$ (15.a)

where,

$P = \frac{\phi}{F}$ = permeance of the magnetic circuit.

$$W'_f = \int_0^i \lambda\, di$$

$$W_f = \int_0^i Li\, di$$

$$W_f = \frac{1}{2} L i^2$$

$W'_f(i, x) = \frac{1}{2} L(x)\, i^2 \longrightarrow$ (16.a)

Expression for coenergy density

$$W'_f = \int_0^H \frac{1}{2}\mu H^2$$

$\to W_f = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2}\frac{\mu^2 H^2}{\mu} = \frac{1}{2}\mu H^2$

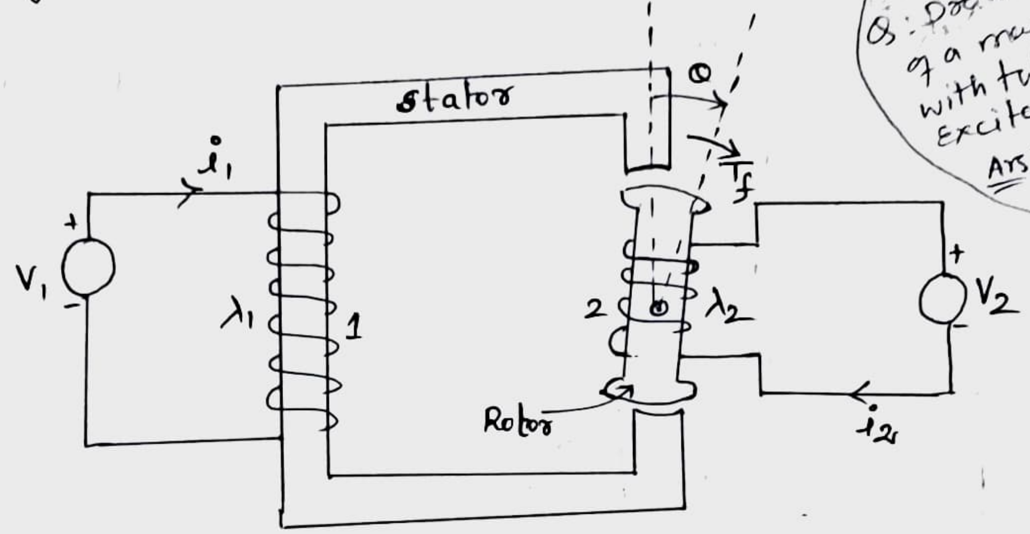
which for the linear case becomes

$W'_f = \frac{1}{2}\mu H^2 = \frac{1}{2}\, \frac{B^2}{\mu} \longrightarrow$ (17.a)

Question:
In a linear case system prove that field energy and Co. energy are equal.

MULTIPLY – EXCITED MAGNETIC FIELD SYSTEM.

⇒ Continuous energy conversion devices – motor and generator require multiple excitation.



Q: Draw the diagram of a magnetic system with two electrical Excitations.
Ans: Draw this diagram.

let,

1, 2 ⇒ Coil wound on the stator and rotor respectively.
V_1, V_2 ⇒ Excitation voltages supplied to the coil 1 & 2 respectively.
i_1, i_2 ⇒ Current through the Coil 1 & 2 respectively.
λ_1, λ_2 ⇒ flux linkages developed in the coil 1 & 2 respectively.

⇒ The above figure shows a magnetic field system with two electrical excitations – one on stator and the other on rotor.

⇒ The system can be described in either of the two sets of three independent variables: $(\lambda_1, \lambda_2, \theta)$ or (i_1, i_2, θ).

⇒ In terms of the first set

$$T_f = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta}$$

Note:
$$T_f = F_f \, \partial \theta$$
$$F_f = -\frac{\partial W_f(\lambda, x)}{\partial x}$$

where the field energy is given by

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 \, d\lambda_1 + \int_0^{\lambda_2} i_2 \, d\lambda_2 \longrightarrow ①$$

Analogy to eqn $i = \frac{\partial W_f(\lambda, x)}{\partial \lambda}$

$$i_1 = \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1}$$

$$i_2 = \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2}$$

Assuming linearity.

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \longrightarrow ②$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \; ; \; (L_{12} = L_{21}) \longrightarrow ③$$

solving for i_1 and i_2 in terms of λ_1, λ_2 and substituting in eqn ①

$$i_1 = \beta_{11} \lambda_1 + \beta_{12} \lambda_2$$

$$i_2 = \beta_{21} \lambda_1 + \beta_{22} \lambda_2 \; ; \quad \beta_{21} = \beta_{12}$$

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 d\lambda_1 + \int_0^{\lambda_2} i_2 d\lambda_2$$

$$= \int_0^{\lambda_1} (\beta_{11}\lambda_1 + \beta_{12}\lambda_2) d\lambda_1 + \int_0^{\lambda_2} (\beta_{21}\lambda_1 + \beta_{22}\lambda_2) d\lambda_2$$

$$= \int_0^{\lambda_1} \beta_{11}\lambda_1 d\lambda_1 + \beta_{12}\lambda_2 d\lambda_1 + \int_0^{\lambda_2} \beta_{21}\lambda_1 d\lambda_2 + \beta_{22}\lambda_2 d\lambda_2$$

$$= \beta_{11} \int_0^{\lambda_1} \lambda_1 d\lambda_1 + \beta_{12}\lambda_2 d\lambda_1 + \beta_{\cancel{22}\,21} \int_0^{\lambda_2} \cancel{\beta_{22}} \lambda_1 d\lambda_2 + \overset{\beta_{22}}{\wedge} \lambda_2 d\lambda_2$$

$$= \beta_{11} \int_0^{\lambda_1} \lambda_1 d\lambda_1 + \beta_{12} \left[\int_0^{\lambda_1} \lambda_2 d\lambda_1 + \int_0^{\lambda_2} \lambda_1 d\lambda_2 \right] + \beta_{22} \int_0^{\lambda_2} \lambda_2 d\lambda_2$$

$$= \beta_{11} \int_0^{\lambda_1} \lambda_1 d\lambda_1 + \beta_{12} \int_0^{\lambda_1, \lambda_2} d(\lambda_1 \lambda_2) + \beta_{22} \int_0^{\lambda_2} \lambda_2 d\lambda_2$$

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2} \beta_{11} \lambda_1^2 + \beta_{12} \lambda_1 \lambda_2 + \frac{1}{2} \beta_{22} \lambda_2^2 \longrightarrow ④$$

where,

$$\beta_{11} = \frac{L_{22}}{(L_{11} L_{22} - L_{12}^2)}$$

$$\beta_{22} = \frac{L_{11}}{(L_{11} L_{22} - L_{12}^2)}$$

$$\beta_{12} = \beta_{21} = -\frac{L_{12}}{(L_{11} L_{22} - L_{12}^2)}$$

The self- and mutual- Inductance of the two exciting coils are functions of angle θ.

If currents are used to describe the system state,

Torque $\rightarrow$
$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

where the co-energy is given by

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1 \, di_1 + \int_0^{i_2} \lambda_2 \, di_2$$

In the linear case

$$W_f'(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

where Inductance are function of angle θ.

The magnitude of magnetic force F_e can be obtained by adopting the procedure as followed magnetic torque.

$$F_e = \frac{1}{2} i_1^2 \frac{dL_{11}}{dx} + \frac{1}{2} i_2^2 \frac{dL_{22}}{dx} + i_1 i_2 \frac{dM_{12}}{dx}$$

$$F_e = \frac{\partial W_f}{\partial x} (i_1, i_2, x)$$

$$F_e = \frac{\partial W_f}{\partial x} (i_1, i_2, x)$$

The force act in such direction as to tend to increase the field energy at constant currents.

14. A coil of 1500 turns carrying a current of 5 Amps produces a flux of 2.5mWb. Find the self inductance of the coil. (NOV/DEC, 2012)

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{1500 \times 2.5 \times 10^{-3}}{5} = 0.75 \text{ Hendri.}$$

---

15. A conductor 80 cm long moves at right angle to its length at a constant speed of 30 m/s in a uniform magnetic field of flux density 1.2T. Find the emf induced when the conductor motion is normal to the field flux. (Apr/may 2011)

$e = Blv \sin\theta$

$e = 1.2 \times 0.8 \times 30 \times \sin 90°$

$e = 28.8$ volts.

---

16. Define staking factor. many magnetic cores used in electronics transformers and inductors are tape-wound from very thin strips of magnetic material. Laminating a magnetic part usually increases its volume.

Refer electrical power [illegible] Pg: 34

The net cross-sectional area of the core occupied by the magnetic material is less than its gross (total) cross-sectional; The ratio (less than unity) being known as staking factor.

Depending upon the thickness of laminations, staking factor may vary from 0.5 - 0.95, approaching unity as the lamination thickness increases.

$$\text{staking factor} = \frac{\text{Actual}}{\text{Gross}}$$

1∞ lamination

Eg: 1mm

Gross = 100mm

Iron = 95mm

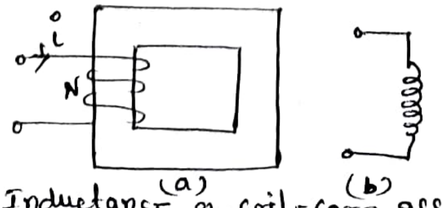
$$\text{staking factor} = \frac{95}{100} = 0.95$$

---

17. Define inductance.

A coil wound on a magnetic core, as shown below in fig(a) is frequently used in electric circuits. This coil may be represented by an ideal circuit element, is called inductance.

$$\boxed{L = \frac{\lambda}{i}} \text{ Hendri}$$



Inductance of coil-core assembly

(a) coil-core assembly (b) Equivalent inductance.

7. Name the main magnetic quantities with their symbols having the following units : webers , Tesla, AT/Wb , H/m. (NOV/DEC, 2013)

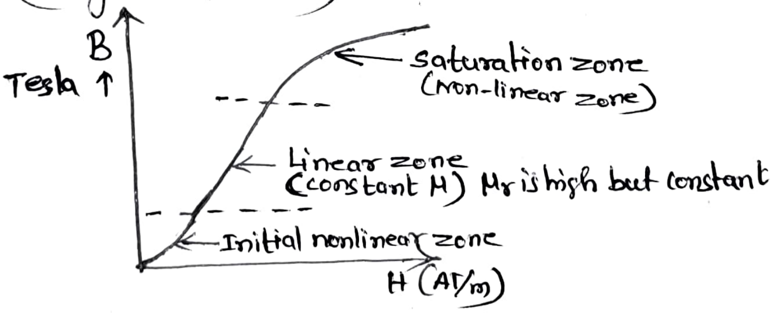
flux ϕ in webers.

magnetic flux density B in Tesla.

Reluctance $\mathcal{R}$ in AT/Wb

Inductance per metre length 'L' in H/m.

8. Draw the typical magnetization curve of ferromagnetic material. (may / Jun, 2013)



14. A coil of 1500 turns carrying a current of 5 Amps produces a flux of 2.5mWb. Find the self inductance of the coil.
(NOV/DEC, 2012)

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{1500 \times 2.5 \times 10^{-3}}{5} = 0.75 \text{ Hendri.}$$

---

15. A conductor 80 cm long moves at right angle to its length at a constant speed of 30 m/s in a uniform magnetic field of flux density 1.2T. Find the emf induced when the conductor motion is normal to the field flux. (Apr/may 2011)

$$e = Blv \sin\theta$$
$$e = 1.2 \times 0.8 \times 30 \times \sin 90^\circ$$
$$e = 28.8 \text{ volts.}$$

---

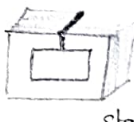
16. Define staking factor. many magnetic cores used in electronics transformers and inductors are tape-wound from very thin strips of magnetic material. Laminating a magnetic part usually increases its volume.

Refer electronic mle Pom Pitam Sayed Nalar P8:34

The net cross-sectional area of the core occupied by the magnetic material is less than its gross (Total) cross-sectional. The ratio (less than unity) being known as staking factor.

Depending upon the thickness of laminations, staking factor may vary from 0.5 – 0.95, approaching unity as the lamination thickness increases.

$$\text{staking factor} = \frac{\text{Actual}}{\text{Gross}}$$



100 laminations
Eg: 1mm
Gross = 100mm
Iron = 95mm

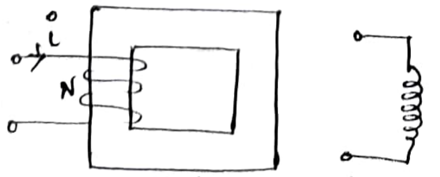
$$\text{Staking factor} = \frac{95}{100} = 0.95$$

---

17. Define inductance.

A coil wound on a magnetic core, as shown below in fig(a) is frequently used in electric circuits. This coil may be represented by an ideal circuit element, is called inductance.

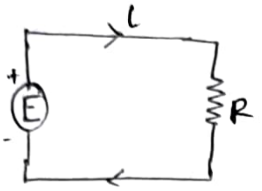
$$\boxed{L = \frac{\lambda}{i}} \text{ Hendri}$$



Inductance of coil-core assembly
(a) coil-core assembly (b) Equivalent inductance.

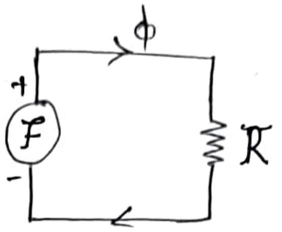
18. Compare between magnetic circuit and Electric circuit. Analogy

Electric circuit



Electric equivalent circuit

Magnetic circuit



Magnetic equivalent circuit.

Driving force	EMF (E) volts.	Driving force	MMF ($\mathcal{F}$)=IN Amperturns
produces	Current $\left(i = \frac{E}{R}\right)$ Amps	produces	Flux $\left(\phi = \mathcal{F}/\mathcal{R}\right)$ webers.
Limited by	Resistance $\left(R = \frac{l}{\sigma A}\right)$		Reluctance $\left(\mathcal{R} = \frac{l}{\mu A}\right)$ $\frac{\text{Amperturns}}{\text{Webers}}$.

---

19. state ohm's law for magnetic circuit.

$$\text{Flux}, \phi = \frac{\mathcal{F}}{R} \text{ webers.}$$

According to ohm's law the flux ϕ is directly propotional to the force(Ni) and inversly propotional to the Reluctance 'R'.

---

20 Distinguish between statically induced and dynamically induced e.m.f's.

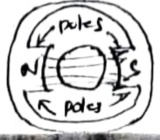
statically induced emf:→ The coil remains stationary with respect to flux, but the flux through it changes with time. The emf induced is known as statically induced emf. $\left(e = L\frac{di}{dt}\right)$

Dynamically induced emf:→ Flux density distribution remains constant and stationary but the coil moves relative to it, the emf induced is known as dynamically induced or motional emf. Eg: machine (rotating m/c)

---

21. what is meant by composite structure?

A magnetic circuit having two or more media - such as the magnetic core and air gap is known as composite structure.



problem: ②

Apr/May 2011 (6 mark)

① The magnetic flux density on the surface of an iron face is 1.6 T which is typical saturation level value for ferromagnetic material. Find the force density on the iron face.

Apr/May 2008 (6 mark)

Solution:

Area of the iron face be $A(m)^2$, consider the field energy in the volume contained between the two faces with a normal distance x.

$$W_f(B,x) = \frac{1}{2}\frac{B^2 Ax}{\mu}$$

The mechanical force due to the field is

$$F_f = -\frac{\partial W_f(B,x)}{\partial x} = -\frac{1}{2}\frac{B^2 A}{\mu}$$

The negative sign indicates that the force acts in a direction to reduce x (i.e. it is an attractive force between the two faces).

The force per unit area is

$$|F_f| = \frac{1}{2}\frac{B^2}{\mu}$$

$$= \frac{1}{2} \times \frac{(1.6)^2}{4\pi \times 10^{-7}}$$

$\mu = 4\pi \times 10^{-7}$

$$= 1.01859 \times 10^6 \text{ N/m}^2$$

(or)

$$= 1018591.6 \text{ N/m}^2$$

③ Two coupled coils have self-and mutual-inductance of

Apr/may 2010

$$L_{11} = 2 + \frac{1}{2x} \; ; \; L_{22} = 1 + \frac{1}{2x} \; ; \; L_{12} = L_{21} = \frac{1}{2x}$$

over a certain range of linear displacement x. The first coil is excited by a constant current of 20 A and the second by a constant current of −10 A. Find

Nov/Dec 2012

in this sem

(a) Mechanical ~~supplied~~ work done if x changes from 0.5 to 1m.

(b) Energy supplied by each electrical source is part(a)

Apr/May 08

(c) change in field energy in part (a)

a

Hence verify that the energy supplied by the source is equal to the increase in the field energy plus the mechanical work done.

Solution:

Since it is the case of current excitations, the expression of co energy will be used.

$$W'_f(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$W'_f(i_1 i_2, x) = \frac{1}{2}\left(2 + \frac{1}{2x}\right) i_1^2 + \frac{1}{2x} i_1 i_2 + \frac{1}{2}\left(1 + \frac{1}{2x}\right) i_2^2$$

$$= \frac{1}{2}\left(2 + \frac{1}{2x}\right)(20)^2 + \frac{1}{2x}(20)(-10) + \frac{1}{2}\left(1 + \frac{1}{2x}\right)(-10)^2$$

$$= \left(2 + \frac{1}{2x}\right)\left(\frac{1}{2}\right)(400) + \frac{1}{2x}(-200) + \left(1 + \frac{1}{2x}\right)\left(\frac{1}{2}\right)(+100)$$

$$= \left(2 + \frac{1}{2x}\right) \times 200 + \frac{1}{2x} \times (-200) + \left(1 + \frac{1}{2x}\right) \times 50$$

$$400 + \frac{200}{2x} - \frac{200}{2x} + 50 + \frac{50}{2x}$$

$$= 400 + \frac{100}{x} - \frac{100}{x} + 50 + \frac{25}{x}$$

$$W'_f(i_1, i_2, x) = 450 + \frac{25}{x}$$

Take derivative w.r.to x

$$= 0 + 25 \times x^{-1}$$
$$= -1 \times 25 \times x^{-1-1}$$
$$= -25 \times x^{-2}$$
$$= \frac{-25}{x^2}$$

(a) $$F_f = \frac{\partial W'_f}{\partial x} = -\frac{25}{x^2}$$

Ref. eqn 15.6 page 3.12

work done = force × distance

$$\Delta W_m = \int_{0.5}^{1} F_f \, dx = \int_{0.5}^{1} -\frac{25}{x^2} dx = -25 \text{ J}$$

$$\int -\frac{25}{x^2} dx$$
$$= \int -25 x^{-2} dx$$
$$= \left[\frac{-25 x^{-2+1}}{-2+1}\right]_{0.5}^{1}$$
$$= \left[\frac{25}{x}\right]^{1}$$

$$= \left[\frac{25}{1} - \frac{25}{0.5}\right] = 25 - 50$$

3.25

(b) $$\Delta W_{e1} = \int_{\lambda_1(x=0.5)}^{\lambda_1(x=1)} i_1 \, d\lambda_1 = i_1 \left[\lambda_1(x=1) - \lambda_1(x=0.5) \right]$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_1 = \left(2 + \frac{1}{2x}\right) \times 20 + \frac{1}{2x} \times (-10)$$

$$\lambda_1 = 40 + \frac{10}{x} - \frac{5}{x}$$

$$\lambda_1 = 40 + \frac{5}{x}$$

$x = 0.5$: (i) $40 + \frac{5}{0.5}$, $40 + 10$, $\lambda_1 = 50$

$x = 1$: (ii) $40 + \frac{5}{1}$, $40 + 5$, $\lambda_1 = 45$

$$\lambda_1(x=0.5) = 50, \quad \lambda_1(x=1) = 45$$

$$\therefore \quad \Delta W_{e1} = 20(45 - 50) = -100 \text{ J}$$

$\Delta W_{e1} = \int_{\lambda_1(x=0.5)}^{\lambda_1(x=1)} i_1 \, d\lambda_1 = i_1[\lambda_1(x=1) - \lambda_1(x=0.5)]$
$= 20(45 - 50)$
$= -100$ J

|||ly

$\Delta W_{e2} = \int_{\lambda_2(x=0.5)}^{\lambda_2(x=1)} i_2 \, d\lambda_2 = i_2[\lambda_2(x=1) - \lambda_2(x=0.5)]$

$$\Delta W_{e2} = i_2 \left[\lambda_2(x=1) - \lambda_2(x=0.5) \right]$$

$$\lambda_2 = L_{12} i_1 + L_{22} i_2$$

$$\lambda_2 = \frac{1}{2x} \times 20 + \left(1 + \frac{1}{2x}\right) \times (-10)$$

$$\lambda_2 = \frac{10}{x} - 10 - \frac{5}{x}$$

$$\lambda_2 = -10 + \frac{5}{x}$$

$\lambda_2 = \frac{10}{2x} + \left(1 + \frac{1}{2x}\right) \times (-10)$
$= \frac{10}{x} + \left(-10 - \frac{10}{2x}\right)$
$= \frac{10}{x} - 10 - \frac{10}{2x}$
$= \frac{10}{x} - 10 - \frac{5}{x}$
$= -10 + \frac{5}{x}$

$\lambda_2 = -10 + \frac{5}{x}$
$x = 0.5$: $-10 + \frac{5}{0.5}$, $-10 + 10$, 0
$x = 1$: $-10 + \frac{5}{1}$, $= -10 + 5$, $= -5$

$$\lambda_2(x=0.5) = 0, \quad \lambda_2(x=1) = -5$$

$$\Delta W_{e2} = -10(-5) = 50 \text{ J}$$

$\Delta W_{e2} = i_2[\lambda_2(x=1) - \lambda_2(x=0.5)]$
$= -10[-5 - 0] = 50$

Net electrical energy input, $\Delta W_e = \Delta W_{e1} + \Delta W_{e2}$
$= -100 + 50$
$= -50$ J

(i) − 0
0 − 5

(c) For calculating the change in the field energy, β's have to be obtained

(β_{11}, β_{22} & $\beta_{12} = \beta_{21}$) ← Refer for the double excited system.

$$\beta_{11} = \frac{L_{22}}{D} \; ; \; D = L_{11}L_{22} - L_{12}^2$$

$$= \frac{2x+1}{4x+3}$$

$$\beta_{11} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2}, \quad \beta_{22} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2}$$

$$\beta_{12} = \beta_{21} = \frac{-L_{12}}{L_{11}L_{22} - L_{12}^2}$$

IIIly

$$\beta_{22} = \frac{L_{11}}{D}$$

$$\beta_{22} = \frac{4x+1}{4x+3}$$

$$\beta_{12} = \beta_{21} = \frac{-L_{12}}{D}$$

$$\beta_{12} = -\frac{1}{4x+3}$$

$$\underbrace{L_{11}L_{22} - L_{12}^2}_{D} = \left(2+\frac{1}{2x}\right)\left(1+\frac{1}{2x}\right) - \left(\frac{1}{2x}\right)^2$$

$$= 2 + \frac{1}{2x} + \frac{1}{x} + \cancel{\frac{1}{4x^2}} - \cancel{\frac{1}{4x^2}}$$

$$= 2 + \frac{1.5}{x} = \frac{2x+1.5}{x}$$

$$\therefore \beta_{11} = \frac{\left(1+\frac{1}{2x}\right)}{\left(\frac{2x+1.5}{x}\right)} = \frac{2x+1}{4x+3}$$

$$\beta_{22} = \frac{\left(2+\frac{1}{2x}\right)}{\left(\frac{2x+1.5}{x}\right)} = \frac{4x+1}{4x+3}$$

$$\beta_{12} = \frac{-\frac{1}{2x}}{\left(\frac{2x+1.5}{x}\right)} = -\frac{1}{4x+3}$$

At $x = 0.5$; $\beta_{11} = \frac{2}{5}$, $\beta_{22} = \frac{3}{5}$, $\beta_{12} = -\frac{1}{5}$

At $x = 1$; $\beta_{11} = \frac{3}{7}$; $\beta_{12} = -\frac{1}{7}$

The value of λ have already been calculated at $x = 0.5, 1$m.

$\lambda_1 (\text{at } x = 0.5) = 50$, $\lambda_1 (\text{at } x = 1) = 45$

$\lambda_2 (\text{at } x = 0.5) = 0$, $\lambda_2 (\text{at } x = 1) = -5$

As per eqn ② (below), The field energy is given by

$$W_f = \frac{1}{2}\beta_{11}\lambda_1^2 + \beta_{12}\lambda_1\lambda_2 + \beta_{22}\lambda_2^2$$

$W_f(\lambda_1, \lambda_2, x) = \frac{1}{2}$ [illegible]

$\lambda_2 = 0$ so W_f [illegible]

The field energy at $x = 0.5$m and $x = 1$ m is then calculated as

$$W_f(x=0.5) = \frac{1}{2} \times \frac{2}{5} \times (50)^2 + \beta_{12}\lambda_1\lambda_2 + \beta_{22}\lambda_2^2$$

↑ The other term become zero because (λ_2 = 0 at $x = 0.5$)

$$= 500 \text{ J}$$

$$W_f(x=1) = \frac{1}{2} \times \frac{3}{7} \times (45)^2 - \frac{1}{7} \times 45 \times (-5) + \frac{1}{2} \times \frac{5}{7} \times (-5)^2$$

$$= 475 \text{ J}$$

Hence

$$\Delta W_f = W_f(x=1) - W_f(x=0.5)$$

$$= 475 - 500$$

$$= -25 \text{ J}$$

← already find out

$$\Delta W_f + \Delta W_m = -25 - 25$$

$$= -50 \text{ J} = \Delta W_e \text{ (verified)}$$

Note!: In the linear case with constant current excitation

$$\Delta W_f = \Delta W_f'$$

ΔW_f can be easily calculated from part (a) without the need of calculating β's. Thus

$$W_f' = 450 + \frac{25}{x}$$

$$\Delta W_f' = W_f'(x=1) - W_f'(x=0.5)$$

$$= 475 - 500$$

$$= -25J = \Delta W_f'$$

(at $x=1$)

$$W_f' = 450 + \frac{25}{x}$$

$$= 450 + \frac{25}{1}$$

$$= 450 + 25$$

$$W_f' = 475$$

Illy at ($x=0.5$)

$$W_f' = 450 + \frac{25}{x}$$

$$= 450 + \frac{25}{0.5}$$

$$= 450 + 50$$

$$= 500$$

Nov/Dec 2012 (16 marks)

④ Two coupled coils have self - and mutual - inductance

$L_{11} = 2 + \frac{1}{2x}$; $L_{22} = 1 + \frac{1}{2x}$; $L_{21} = \frac{1}{2x}$

Find the expression for the time - average force of field origin at $x = 0.5$ m if

(a) both coils are connected in parallel across a voltage source of 100 cos 314t V,

(b) both coils are connected in series across a voltage source of 100 cos 314t V.

(c) coil 2 is shorted and coil 1 is connected to a voltage source of 100 cos 314t V, and

(d) both coils are connected in series and carry a current of 0.5 cos 314tA

$\frac{d(x^n)}{dx} \to n x^{n-1}$

$\frac{1}{x} = x^{-1}$

$\frac{d}{dx}(c) \to 0$

$(x^{-1}) \to (-1) x^{-2}$

Solution:

$$W_f'(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$W_f'(i_1, i_2, x) = \underbrace{\frac{1}{2}\left(2 + \frac{1}{2x}\right) i_1^2}_{\frac{1}{4} x^{-1} i_1^2} + \left(\cancel{1+}\frac{1}{2x}\right) i_1 i_2 + \frac{1}{2}\left(\frac{1}{2x}\right) i_2^2$$

$\frac{\partial}{\partial x} \frac{1}{2}\left(2 + \frac{1}{2x}\right) = \frac{\cancel{2}1}{\cancel{2}} + \frac{1}{4x}$

$$F_f = \frac{\partial W_f'(i_1, i_2, x)}{\partial x} = \frac{\partial}{\partial x}\left[\left(1 + \frac{1}{4x}\right) i_1^2 + \left(1 + \frac{1}{2x}\right) i_1 i_2 + \frac{1}{4x} i_2^2\right]$$

$$= \frac{1}{4}(-1) x^{-1-1} i_1^2 + \frac{1}{2}(-1) x^{-1-1} i_1 i_2 + \frac{1}{4}(-1) x^{-1-1} i_2^2$$

$$= -\frac{1}{4} x^{-2} i_1^2 - \frac{1}{2} x^{-2} i_1 i_2 - \frac{1}{4} x^{-2} i_2^2$$

$$= -\frac{1}{4x^2} i_1^2 - \frac{1}{2x^2} i_1 i_2 - \frac{1}{4x^2} i_2^2$$

$\frac{\partial}{\partial x}\left(1 + \frac{1}{4x}\right)$

$= 0 + \frac{1}{4}(-1) x^{-2}$

$= \frac{-1}{4x^2}$

$\frac{\partial}{\partial x}\left[\frac{1}{2x}\right] = \frac{1}{2}(-1) x^{-2} = \frac{-1}{2x^2}$

$\frac{\partial}{\partial x}\left(\frac{1}{4x}\right) = \frac{1}{4}(-1) x^{-2} = \frac{-1}{4x^2}$

3·29

$\int \frac{di_1}{dt} = 20 \sin 314t$

$i_1 = \frac{20 \cos 314t}{314 (1)}$

$i_1 = \frac{20}{314.} \cos 314t$

For $x = 0.5m$

$$F_f = -\frac{1}{4(0.5)^2} i_1^2 - \frac{1}{2(0.5)^2} i_1 i_2 - \frac{1}{4(0.5)^2} i_2^2$$

$$= -\frac{1}{1} i_1^2 - \frac{1}{0.5} i_1 i_2 - \frac{1}{1} i_2^2$$

$$F_f = -i_1^2 - 2 i_1 i_2 - i_2^2$$

The force acts in a direction to decrease x.

(a) Both coils connected in parallel across the voltage source:

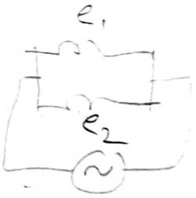
$$L_{11} = 2 + \frac{1}{2x}\Big|_{x=0.5} = 3$$

$$\lambda_1 = L_{11} i_1 + M i_2$$

$$L_{22} = 1 + \frac{1}{2x}\Big|_{x=0.5} = 2$$

$$\lambda_2 = L_{22} \cdot i_2 + M i_1$$

$$M = L_{12} = L_{21} = \frac{1}{2x}\Big|_{x=0.5} = 1$$



$$v = e_1 = \frac{d\lambda_1}{dt} = 3 \frac{di_1}{dt} + 1 \times \frac{di_2}{dt} = 100 \cos 314t \rightarrow ①$$

$$v = e_2 = \frac{d\lambda_2}{dt} = 1 \times \frac{di_1}{dt} + 2 \frac{di_2}{dt} = 100 \cos 314t \rightarrow ②$$

$① \times 2 \Rightarrow 6 \frac{di_1}{dt} + 2 \frac{di_2}{dt} = 200 \cos 314t$

$② \times 1 \Rightarrow \frac{di_1}{dt} + 2 \frac{di_2}{dt} = 100 \cos 314t$

$(-) \qquad (-) \qquad (-)$

$5 \frac{di_1}{dt} = 100 \cos 314t$

Solving we get

$\frac{di_1}{dt} = \frac{\cancel{100}^{20}}{\cancel{5}} \cos 314t$

$$\frac{di_1}{dt} = 20 \cos 314t$$

$\int di_1 = 20 \cos 314t \, dt$

$i_1 = \frac{20 \sin 314t}{314}$

$$\frac{di_2}{dt} = 40 \cos 314t$$

$① \times 1 \Rightarrow 3 \frac{di_1}{dt} + \frac{di_2}{dt} = 100 \cos 314t$

$② \times 3 \Rightarrow 3 \frac{di_1}{dt} + 6 \frac{di_2}{dt} = 300 \cos 314t$

$(-) \qquad (-) \qquad (-)$

$-5 \frac{di_2}{dt} = -200 \cos 314t$

$\frac{di_2}{dt} = \frac{200}{5} \cos 314t$

$= 40 \cos 314t$

Integrating

$$i_1 = \frac{20}{314} \sin 314t$$

$$i_2 = \frac{40}{314} \sin 314t$$

Note $M = 1$
$\uparrow$
mutual inductance

$e = \frac{d\lambda}{dt}$

$e_1 = \frac{d\lambda_1}{dt}$

$\lambda_1 = L_{11} i_1 + M i_2$

$e_1 = \frac{d\lambda_1}{dt} = 3 \frac{di_1}{dt} + 1 \frac{di_2}{dt} \rightarrow ①$

$e_2 = \frac{d\lambda_2}{dt} =$

$\lambda_2 = L_{22} i_2 + M i_1$

$e_2 = \frac{d\lambda_2}{dt} = 2 \frac{di_2}{dt} + 1 \frac{di_1}{dt}$

$e_2 = \frac{d\lambda_2}{dt} = 1 \frac{di_1}{dt} + 2 \frac{di_2}{dt} \rightarrow ②$

Substituting for i_1 and i_2 in the expression for F_f,

$$F_f = -i_1^2 - 2i_1 i_2 - i_2^2$$

$$F_f = -\left(\frac{20}{314}\right)^2 \sin^2 314t - 2\left(\frac{20}{314}\right)\left(\frac{40}{314}\right)\sin 314t \sin 314t - \left(\frac{40}{314}\right)^2 \sin^2 314t$$

$$= -\frac{1}{(314)^2}\left[(20)^2 + 2 \times 20 \times 40 + (40)^2\right] \sin^2(314t)$$

$$F_f = -\frac{1}{(314)^2}\left[(20)^2 + 2 \times 20 \times 40 + (40)^2\right] \sin^2 314t$$

$$F_f = -\frac{1}{(314)^2}\left[3600\right] \sin^2 314t$$

$$\boxed{3\cancel{600} \quad 60^2}$$

$$F_f = -\frac{1}{(314)^2}\left[60^2\right] \sin^2 314t$$

$$F_f = -\left(\frac{60}{314}\right)^2 \sin^2 314t \qquad F_f = -\left(\frac{60}{314}\right)^2 \sin^2 314t.$$

$\sin^2 \omega t$

$\omega = 2\pi f$

$\omega = 314$

$\sin^2 314t$

But $$\frac{1}{T}\int_0^T \sin^2 \omega t \, dt = \frac{1}{2}$$

$$\frac{1}{2}(t)_0^T - \frac{1}{2}\left(\frac{\sin 2\omega t}{2\omega}\right)_0^T = \frac{T}{2} - \frac{1}{2\omega}\left[\sin 2\omega T\right] \cancel{\sin} =$$

$$\therefore \quad F_f(av) = -\frac{1}{2}\left(\frac{60}{314}\right)^2$$

$$F_f(av) = -0.0183 \text{ N}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$\sin^2\theta = [1 - \cos 2\theta]/2$$

(b) Both coils connected in series across the voltage source:

$$v = \frac{d\lambda_1}{dt} + \frac{d\lambda_2}{dt}$$

$$= \left(3\frac{di_1}{dt} + \frac{di_2}{dt}\right) + \left(\frac{di_1}{dt} + 2\frac{di_2}{dt}\right)$$

But $i_1 = i_2 = i$ (Series connection)

$$\therefore \quad v = 7\frac{di}{dt} = 100 \cos 314t$$

$$\int 7 di = \int 100 \cos 314t \, dt$$

$$i = \frac{100 \sin 314t}{7 \times 314}$$

Integrating we get

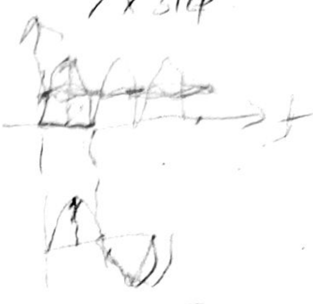
$$i = \frac{100}{7 \times 314} \sin 314t$$

$\omega t = 0$ 3.31 ~~3.29~~

Substituting in the expression for F_f,

$$F_f = -\left(\frac{100}{7\times314}\right)^2 \sin^2\omega t - 2\left(\frac{100}{7\times314}\right)^2 \sin^2\omega t - \left(\frac{100}{7\times314}\right)^2 \sin^2\omega t$$

$$F_f = -4 \times \left(\frac{100}{7\times314}\right)^2 \sin^2 \omega t$$



(or) $F_f\,(av) = -2\left(\frac{100}{7\times314}\right)^2$

$F_f\,(av) = -0.00144$ N
(or)
-0.004139

RMS $= \frac{I_{max}}{\sqrt{2}}$

(c) Coil 2 shorted, coil 1 connected to voltage source:

$$100 \cos 314t = 3\frac{di_1}{dt} + 2\frac{di_2}{dt} \quad ①$$

$$0 = \frac{di_1}{dt} + 2\frac{di_2}{dt} \quad ②$$

2① − ② ⇒

$200 \cos 314t = 6\frac{di_1}{dt} + 2\frac{di_2}{dt}$

$-\;0 = \frac{di_1}{dt} - 2\frac{di_2}{dt}$

$200 \cos 314t = 5\frac{di_1}{dt}$

$\frac{di_1}{dt} = \frac{~~200~~\,40}{~~5~~} \cos 314t$

$= 40 \cos 314t$

$\frac{di_1}{dt} = 40 \cos 314t$

$\int di_1 = \int 40 \cos 314t \, dt$

$i_1 = \frac{40 \sin 314t}{314}$

3② − ① ⇒

$0 = 3\frac{di_1}{dt} + 6\frac{di_2}{dt}$

$(-)100\cos 314t = 3\frac{di_1}{dt} + \frac{di_2}{dt}$

$-100\cos 314t = 5\frac{di_2}{dt}$

$\frac{di_2}{dt} = \frac{-~~100~~\,20}{~~5~~}\cos 314t$

$\frac{di_2}{dt} = -20 \cos 314t$

$\int di_1 = \int -20 \cos 314t$

$i_1 = \frac{-20 \sin 314t}{314}$

$i_1 = \frac{-20}{314} \sin 314t$

$\frac{1}{\pi}\int_0^\pi K\sin^2\theta \, d\theta$

$= \frac{K}{\pi}\int_0^\pi \frac{1-\cos 2\theta}{2} \, d\theta$

$= \frac{K}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^\pi$

$= \frac{K}{2}$

Solving we have

$$\frac{di_1}{dt} = 40 \cos 314t$$

$$\frac{di_2}{dt} = -20 \cos 314t$$

upon integrating we get

$$i_1 = \frac{40}{314} \sin 314t$$

$$i_2 = -\frac{20}{314} \sin 314t$$

Substituting for i_1 and i_2 in the expression for F_f

$F_f = -i_1^2 - 2i_1i_2 - i_2^2$

$$F_f = -\frac{1}{314^2}\left[(40)^2 - 2\times40\times20 + (20)^2\right] \sin^2 314t$$

$$F_f\,(av) = -\frac{1}{2}\left(\frac{20}{314}\right)^2 = -0.00203 \text{ N}$$

(d) Both coils in series carrying current:

$$i = 0.5 \cos 314t$$

Substituting in the expression for F_f,

$$F_f = -(1+2+1) \times (0.5)^2 \cos^2 314t$$

$$F_f(av) = -0.5N$$

④ Nov / Dec 2013 part (a)

⑤ A double-excited magnetic field system has coil self- and mutual inductance of

Apr / May 2011. (16 mark a&b)

$$L_{11} = L_{22} = 2$$

$$L_{12} = L_{21} = \cos\theta$$

where θ is the angle between the axes of the coil

(8 mark) (a) The coil are connected in parallel to a voltage source $v = V_m \sin\omega t$. ~~Determine~~ Derive an expression for the instantaneous torque as a function of the angular position θ. Find therefrom the time-average torque. Evaluate for $\theta = 30°$, $v = 100 \sin 314t$

(8 mark) (b) If coil 2 is shorted while coil 1 carries a current of $i_1 = I_m \sin\omega t$, derive expression for the instantaneous and time-average torques. Compute the value of the time-average torque when $\theta = 45°$ and $i_1 = \sqrt{2} \sin 314t$

(c) In part (b) if the rotor is allowed to move, at what value of angle will it come to rest?

Solution:

$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial\theta}$$

$$T_f = \frac{1}{2}\left(\frac{\partial L_{11}}{\partial\theta}\right) i_1^2 + \left(\frac{\partial L_{12}}{\partial\theta}\right) i_1 i_2 + \frac{1}{2}\left(\frac{\partial L_{22}}{\partial\theta}\right) i_2^2$$

Substituting the value of Inductance,

$$T_f = \frac{1}{2}\left(\frac{\partial(2)}{\partial\theta}\right) i_2^2 + \frac{\partial(\cos\theta)}{\partial\theta} i_1 i_2 + \frac{1}{2}\left(\frac{\partial(2)}{\partial\theta}\right) i_2^2$$

$$= 0 + (-\sin\theta) i_1 i_2) + 0$$

$$T_f = -(\sin\theta) i_1 i_2$$

From circuit equations

$$V_m \cos\omega t = 2\frac{di_1}{dt} + (\cos\theta)\frac{di_2}{dt}$$

$$V_m \cos\omega t = (\cos\theta)\frac{di_1}{dt} + 2\frac{di_2}{dt}$$

Solving these we get

$$\frac{di_1}{dt} = \frac{di_2}{dt} = \frac{V_m \cos\omega t}{(2 + \cos\theta)}$$

Integrating

$$i_1 = i_2 = \frac{V_m \sin\omega t}{\omega(2 + \cos\theta)}$$

Substituting in T_f,

$$T_f = \frac{V_m^2 \sin\theta}{(2 + \cos\theta)^2 \omega^2} \sin^2\omega t$$

$$T_f\,(av) = -\frac{V_m^2 \sin\theta}{2(2+\cos\theta)^2 \omega^2}$$

Given: $\theta = 30^\circ$, $V = 100 \sin 314t$

$$\therefore \quad T_f\,(av) = -\frac{(100)^2 \sin 30^\circ}{2(2+\cos 30^\circ)^2 \times (314)^2}$$

$$= -0.069 \text{ Nm}$$

(b) From circuit equations

$$0 = (\cos\theta)\frac{di_1}{dt} + 2\frac{di_2}{dt}$$

(or) $$\frac{di_2}{dt} = -\frac{1}{2}(\cos\theta)\frac{di_1}{dt}$$

(or) $$i_2 = -\frac{1}{2}(\cos\theta)\, i_1$$

Given: $i_1 = I_m \sin\omega t$

$$\therefore \quad i_2 = -\frac{1}{2} I_m (\cos\theta) \sin\omega t$$

Substituting in T_f

$$T_f = -(\sin\theta) \times \frac{1}{2} I_m^2 (\cos\theta) \sin^2\omega t$$

$$= -\frac{1}{2} I_m^2 (\sin\theta)(\cos\theta)\sin^2\omega t$$

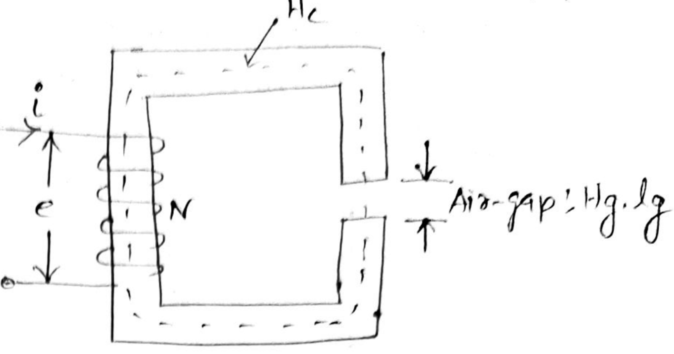
$$T_f\,(av) = -\frac{1}{8} I_m^2 (\sin 2\theta)$$

Given: $\theta = 45^\circ$, $I_m = \sqrt{2}$

$$\therefore \quad T_f\,(av) = \frac{1}{8} \times 2 \sin 90^\circ = 0.25 \text{ Nm}$$

(c) The average torque is zero and changes sign at $\alpha = 0°$, $90°$, $180°$. The rotor can come to rest at any of these values of α but the position of stable equilibrium will only be $\alpha = 90°, 270°, \ldots$ (The reader should draw T_f (av) versus α and reason out).

2. The magnetic circuit has dimensions: $A_c = 4\times4$ cm², $L_g = 0.06$cm, $L_c = 40$cm and N = 600 turns. Assume the value of $\mu_r = 6000$ for iron. Find the exciting current for $B_c = 1.2$ T and the corresponding flux and flux linkages. **(16)**



$$Ni = \frac{B_c}{\mu_c} l_c + \frac{B_g}{\mu_0} l_g$$

(neglecting fringing) $A_c = A_g \therefore B_c = B_g$

$\mu_c = \mu_0 \mu_r$

$$Ni = \frac{B_c}{\mu_0 \mu_r} l_c + \frac{B_c}{\mu_0} l_g$$

$$Ni = \frac{B_c}{\mu_0}\left(\frac{l_c}{\mu_r} + l_g\right)$$

$$i = \frac{B_c}{\mu_0 N}\left(\frac{l_c}{\mu_r} + l_g\right)$$

$$i = \frac{1.2}{4\pi\times10^{-7}\times600}\left(\frac{40}{6000} + 0.06\right)\times10^{-2}$$

$$i = 1591.549\left(6.666\times10^{-3} + 0.06\right)\times10^{-2}$$

$$i = 1591.549\left(6.6666\times10^{-4}\right)$$

$$\boxed{i = 1.06 \text{ Amps}}$$

$\phi = B_c A_c$

$\phi = 1.2\times16\times10^{-4}$ webers.

$\boxed{\phi = 1.92\times10^{-3} = 19.2\times10^{-4}}$ webers.

$\lambda = N\phi = 600\times19.2\times10^{-4}$ web Turns.

$\boxed{\lambda = 1.152}$ wb-turns.

If fringing is to be taken into account,

$A_g = (4+0.06)\,(4+0.06)$

$A_g = 16.484$ cm²

effective $A_g > A_c$ reduces the air-gap reluctance now.

$$B_g = \frac{\phi}{A_g}$$

$$B_g = \frac{19.2\times10^{-4}}{16.484\times10^{-4}} = 1.165 \text{ Tesla.}$$

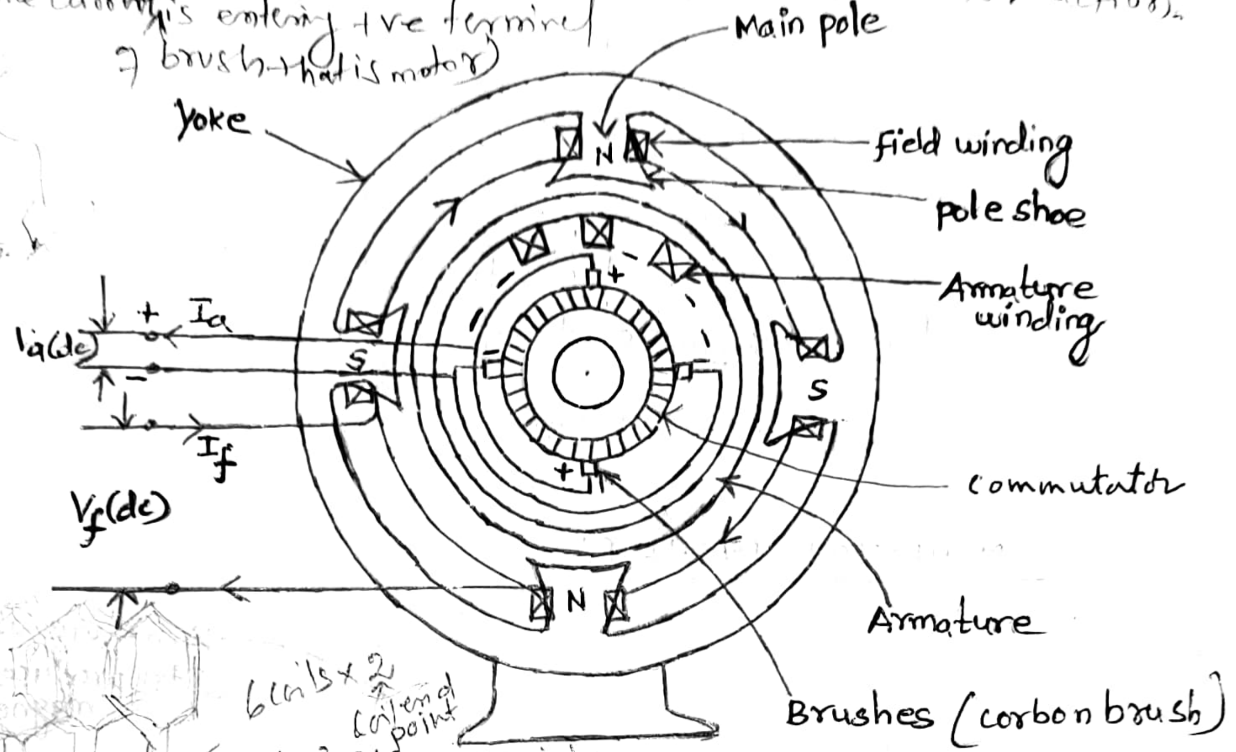
$$i = \frac{1}{\mu_0 N}\left(\frac{B_c l_c}{\mu_r} + B_g l_g\right)$$

$$i = \frac{1}{4\pi\times10^{-7}\times600}\left(\frac{1.2\times40\times10^{-2}}{6000} + 1.165\times0.06\times10^{-2}\right)$$

$$\boxed{i = 1.0345 \text{ A}}$$

D·C machine

note: The current direction away from +ve terminal brush means generator action.
(If the current direction is entering +ve terminal of brush - that is motor)



6 coils × 2 (coil end point)
so 6 × 2 = 12 end points

Cross-sectional view of d·c machine.

12/6 = 6 = No of segments need.

Armature wdg: → Armature winding is nothing but the interconnection of the armature conductors, placed in the slots provided on the armature core periphery. when the armature is rotated, in case of generator, magnetic flux gets cut by armature (conductor & emf) induced in them

→ In a d·c machine the field poles are on the stator while the rotor is the armature as shown in the cross-sectional view in the above fig.

→ The field poles are symmetrical and are even in number, alternately north and south.

→ (commutator ? 2 mark.) As the armature rotates, alternating emf and current induced in the armature winding are rectified to dc form by a rotating mechanical switch called the commutator, which is tapped by means of stationary corbon brushes.

→ The commutator is cylindrical in shape and comprises several wedge-shaped copper segments band together while they are insulated from each other.

It is dividing into two parts (i) Armature core (ii) Armature winding.

→ ① The armature is made of laminated steel with slots cut out on the periphery to accomodate the insulated armature winding.

Armature core → Armature core is cylindrical in shap mounted on the shaft. It is consist of slots on its perihery and the air duct to permit the air flow through armature which serves cooling purposes.

7. b) A 4-pole, lap-wound dc machine has 728 armature conductors. Its field winding is excited from a dc source to create an air-gap flux of 32 mWb/pole. The machine (generator) is run from a prime mover (diesel engine) at 1600 rpm. It supplies a current of 100 A to an electric load.

(i) Calculate the electromagnetic power developed.

Solution:

$$E_a = \frac{\phi n z}{60} \times \left(\frac{P}{A}\right)$$

$$E_a = \frac{32\times10^{-3}\times 1600 \times 728}{60} \times \left(\frac{4}{4}\right)$$

$E_a = 621.2$ volts.

$I_a = 100$ A

Electromagnetic power developed $= E_a I_a$

or

Total Electric power developed $= 621.2 \times 100 = 62{,}120$ watts.

$$= \frac{621 \times 100}{1000} = 62.12 \text{ KW}$$

(ii) What is the mechanical power that is fed from the prime mover to the generator?

Mechanical power provided by prime mover = electromagnetic power developed = 62.12 KW

$$P_m = T\, W_m$$

T → Torque

W_m → Angular speed in rad/sec

(or) Torque of primemover $= \frac{P_m}{W_m}$

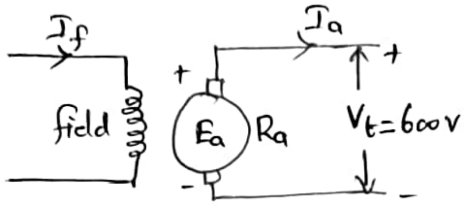
$$= \frac{E_a I_a}{W_m}$$

$$= \frac{62.12 \times 1000}{\left(\frac{2\pi \times 1600}{60}\right)}$$

$= 370.75$ Nm.

8. b) A 1500 Kw, 600V, 16-pole, d.c generator runs at 200 rpm. What must be the useful flux per pole, if there are 2500 lap-connected conductors and full load copper loss are 25Kw? Also calculate the area of the pole-shoe if the average gap flux density is 0.85T.

Solution:



Full-load armature current, $I_a = \frac{\text{o/p power}}{\text{o/p voltage}}$

$$I_a = \frac{1500 \times 1000}{600} = 2500\,A$$

$$\text{Copper-loss} = I_a^2 R_a$$
$$= 25 \times 1000 = 25000.\,W$$

$$R_a = \frac{\text{copper loss}}{I_a^2} = \frac{25000}{2500 \times 2500} = 4 \times 10^{-3}\,\Omega$$

$$E_a = V_t + I_a R_a$$
$$= 600 + 2500 \times 4 \times 10^{-3}$$
$$= 610\,V$$

But $E_a = \frac{\phi n z}{60}\left(\frac{P}{A}\right)$

where, ϕ is the useful flux/pole which links armature coils. Some of the pole flux will coplete its circuit bypassing the armature - called the leakage flux.

Substituting the values,

$$610 = \frac{\phi \times 200 \times 2500}{60}\left(\frac{16}{16}\right)$$

$$\phi = 0.0732\,\text{wb}$$

$$\text{Area of pole-shoe} = \frac{0.0732}{0.85} = 861\,\text{cm}^2$$

PREPARED BY VERIFIED BY APPROVED BY

(d) Armature resistance = 0.2 Ω
(e) Number of conductors = 660
(f) Type of winding—wave
(g) Diameter of the pole shoe = 40 cm
(h) The length of the pole shoe is 25 cm and the pole subtends an angle of 55°
(i) Shunt field current is neglected
(j) Number of poles = 6

Determine the flux density in the air gap.

Solution

$$\text{Load current } = \frac{25000}{240} = 104.17 \text{ A}$$

$$\text{Armature current, } I_a = 104.17 \text{ A}$$

(since the shunt field current is negligible)

$$\begin{aligned}\text{Generated emf} &= V + I_a r_a \\ &= 240 + 104.17 \times 0.2 \\ &= 240 + 20.834 \\ &= 260.834 \text{ V}\end{aligned}$$

$$\begin{aligned}\text{Flux per pole, } \phi_p &= \frac{260.834}{Z \times N \times p/a} \\ &= \frac{260.834}{660 \times \frac{500}{60} \times \frac{6}{2}} = 0.16 \text{W}\end{aligned}$$

$$\begin{aligned}\text{Pole shoe arc } &= \pi D \times \frac{\theta}{360} \\ &= \pi \times 40 \times 10^{-2} \times \frac{55}{360} = 0.192 \text{ m}\end{aligned}$$

$$\text{Pole shoe length} = 25 \times 10^{-2} \text{ m}$$

$$\begin{aligned}\text{Pole shoe area} &= \text{Pole shoe arc} \times \text{Pole shoe length} \\ &= 0.192 \times 25 \times 10^{-2} \\ &= 4.8 \times 10^{-2} = 0.048\end{aligned}$$

$$\text{Flux density in the air gap } = \frac{0.016}{0.048} = 0.33 \text{ Wb/m}^2$$

1.6 ARMATURE REACTION IN DC MACHINES

Now the question arises whether the voltage drop in the dc machine (as discussed in Section 1.5) is on account of the armature resistance alone, or is there any other factor involved as well.

The other factor which is responsible in part for the reduction in voltage of the machine is the *armature reaction*. In other words, there is some reaction from the current-carrying armature conductors which is partly responsible for the drop in the voltage of the machine.

There are two primary mmfs or fluxes operating in the dc machine. One is the field mmf F_m produced by the field windings wound on the N and S poles of the machine. The other is the perpendicular armature flux F_a produced by the current in the armature conductors. These two fluxes are shown in Figure 1.17. The magnetic flux F_a due to the flow of current through the armature conductors *interacts* with the magnetic field F_m of the poles. This interaction causes distortion of the main field flux. This distortion is termed the *cross-magnetization effect*. The resultant sum of F_m and F_a is also shown in Figure 1.17 as the resultant flux F.

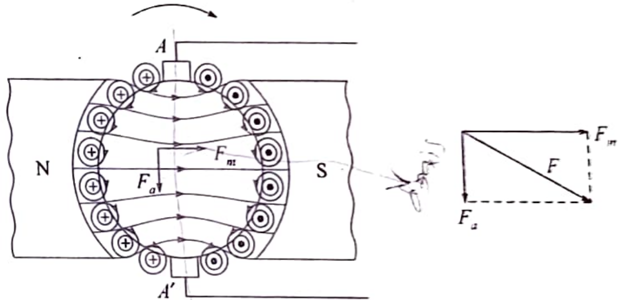
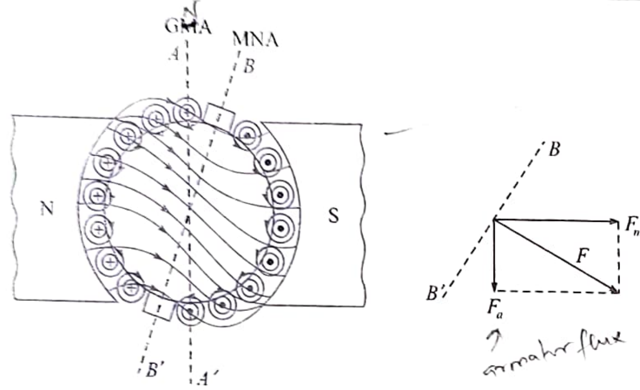


Figure 1.17 Flux distributions of the main magnetic field of the poles and that due to flow of current in armature conductors.

Due to the interaction between F_m and F_a, the field flux entering the armature is not only distorted but also shifted. This shift causes the magnetic neutral axis to be shifted clockwise (e.g. in the direction of rotation of the generator which is also assumed clockwise) to a new position BB'; this new position BB' remains perpendicular to the resultant field flux F. The geometric axis remains at AA'. See Figure 1.18.

The effect of armature reaction therefore causes distortion of the main field flux as well as shifting of the magnetic neutral axis. This effect in turn also causes a reduction in the main field flux. This reduction in main field flux is responsible, in part, for the voltage drop in the machine as discussed in Section 1.5.

Now, if the brushes remain on the original position of the magnetic neutral axis AA' as shown in Figure 1.17, the coil that is being commutated will undergo the greatest change in flux linkage compared to any other coil under the pole, as this position of the magnetic neutral axis (MNA) is now no longer at a point of minimum coil flux. It is therefore obvious that the brushes must be shifted to the new position of the magnetic neutral axis BB' as shown in Figure 1.18, in order to avoid sparking at the brushes due to voltage that will now be induced in the coil if the brushes remain at the position AA'. At the new magnetic neutral axis position BB', the magnetic field due to the armature current will be shifted as shown in Figure 1.19.



main magnetic field (or) field mmf

armature flux

Figure 1.18 The shift of the magnetic neutral axis.

F_a can now be subdivided into two components, i.e. F_d and F_c. F_d is directly opposite to the main magnetic field F_m and F_c creates the same impact as the cross magnetizing field. So, it is observed that by shifting brushes to the actual magnetic neutral axis, some demagnetization effect of the main magnetic field occurs. .

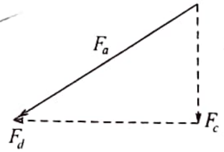


Figure 1.19 Change in direction of armature mmf due to shift of brushes to new MNA.

1.6.1 Calculation of Demagnetization and Cross Magnetization Ampereturns per Pole

① How are its demagnetizing and cross-magnetizing ampere turns calculated? Nov/Dec 2006

The demagnetizing effect and the cross-magnetizing effect due to the shifting of the brushes to the magnetic neutral axis can be mathematical analysed as follows. The conductors lying in the angles BOC and $B'OC'$ develop ampereturns which produce the demagnetizing field, and the conductors lying in the angles COB' and BOC' develop ampereturns which produce the cross-magnetizing field from the point of view of armature reaction (as shown in Figure 1.20).

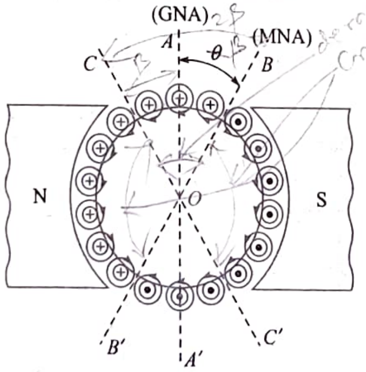


Figure 1.20 Development of demagnetizing field and cross-magnetizing field.

Let us consider the following data:

Z = number of armature conductors

θ = ~~forward lead to~~ magnetic axis from the geometrical axis, in angular degrees

I = current in each armature conductor

The number of armature conductors within the angles BOC and $B'OC'$ is

$$= \frac{4\theta}{360} Z = \frac{4\beta}{360} Z$$

β ⇒ is the brush shift in electrical degrees.

I_a → armature current

$$\text{Total number of turns in the above case} = \frac{2\theta}{360} Z$$

$$\text{Total number of ampereturns in the above case} = \frac{2\theta Z}{360} I$$

Hence, the demagnetizing ampereturns per pair of poles

$$= \frac{2\theta}{360} ZI$$

Therefore, the demagnetizing ampereturns per pole

$$= \frac{\theta}{360} ZI$$

Demagnetizing ampere-turns per pole $= AT_a \times \frac{2\beta}{180}$ AT/pole

The cross-magnetizing conductors per pole

$$= \frac{Z}{p} - \frac{2\theta}{360} Z$$

Hence the cross-magnetizing ampere conductors per pole

$$= ZI\left(\frac{1}{p} - \frac{2\theta}{360}\right)$$

Cross magnetizing ampere-turns per pole $= AT_a\left(1 - \frac{2\beta}{180}\right)$ AT/pole

Therefore, the cross-magnetizing ampereturns per pole

$$= ZI\left(\frac{1}{2p} - \frac{\theta}{360}\right)$$

Note: $\frac{ZI_a}{2AP} = AT_a$

1.6.2 Compensating Winding

① What is the function of compensating winding? 2 mark. Nov/Dec 2012

to neutrolize the armature reaction ampere turns by a compensating winding

Now, the question arises as to how we can eliminate the effect of armature reaction mmf in dc machines. Obviously, some extra arrangement has to be made. This arrangement is nothing but providing a compensating winding on each pole shoe of the dc machine as shown in Figure 1.21. The mode of compensating winding is made such that it opposes the armature reaction ampereturns as far as possible. The compensating winding conductors are connected in series with the armature conductors. Hence, the current which will flow through the compensating winding is the armature current. So the number of turns of the compensating winding as required is designed such that this winding can oppose the armature reaction ampereturns. Hence the number of ampereturns required for the compensating winding per pole, i.e. AT_{CW}/pole will be equal to

$$\text{armature ampereturns} \times \frac{\text{pole arc}}{\text{pole pitch}}$$

$AT_a \times (\;\;\;)$

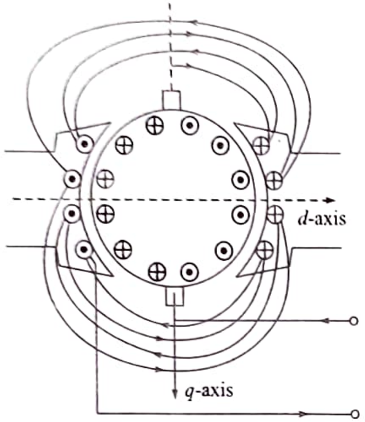
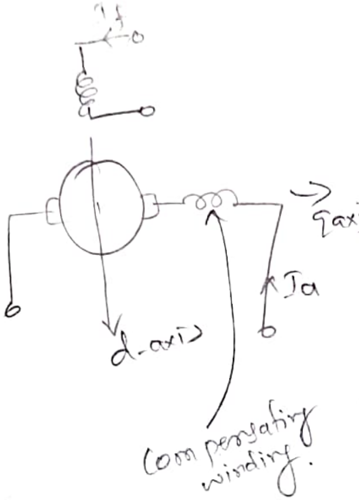


Figure 1.21 Compensating winding.



Hence, (ii) $\text{AT}_{\text{CW}}\text{/pole} = \dfrac{IZ}{2ap} \times \dfrac{\text{pole arc}}{\text{pole pitch}}$

where

a = number of parallel paths through the armature winding
I = current carried by each armature conductor.

But the problem is that the compensating winding neutralizes only the armature reaction mmf directly under the pole. In the interpolar zone, normally no compensation is made as the effect of armature reaction in this zone is negligible. However, to make the system foolproof, we should have an interpole in the interpolar region as well. It is advisable to use a compensating winding in each interpolar region too, specially so in large machines where heavy currents may occur. Also in case of heavy load fluctuations in dc motors, it is advisable to provide compensating windings in the interpole regions as well.

1.6.3 Commutation

The word commutation means the changes in current that take place in a coil during the period of its short circuit by a brush. When the conductors under the influence of north pole come under the influence of south pole, the direction of the current through the conductors will reverse. In Figures 1.22(a) to 1.22(e) the whole process of change in coil current from +30 A to –30 A is described. The described coil Q is undergoing the process of commutation. Figure 1.22(a) shows the brush position on segment q of the commutator. The current in coils P and Q is 30 A each and in coil R it is also 30 A but in the opposite direction. The current through the brush is 60 A. In Figure 1.22(b) the commutator has moved a small distance so that the brush is on segments p and q, thus partially shorting the armature coil Q. The current from coil P begins to enter the commutator segment p, reducing the current in coil Q. In Figure 1.22(c) the coil Q is fully short circuited by the brush; consequently no current flows in coil Q. The current in coils P and R is each 30 A in opposite directions. The current through the brush is 60 A. In Figure 1.22(d) the commutator has moved still further and the brush still short circuits the coil Q, this time setting up a current in coil Q in the opposite direction. In Figure 1.22(e), when the brush is on the commutator segment p, the coil Q should be carrying the full current of 30 A in the opposite direction; it, however, carries only, say, 25 A due to an emf of self-induction (as per Lenz's law) that opposes the sudden reversal of current in coil Q. This emf of self-induction is called the *reactance voltage* because it reacts against the current reversal in each coil undergoing commutation.

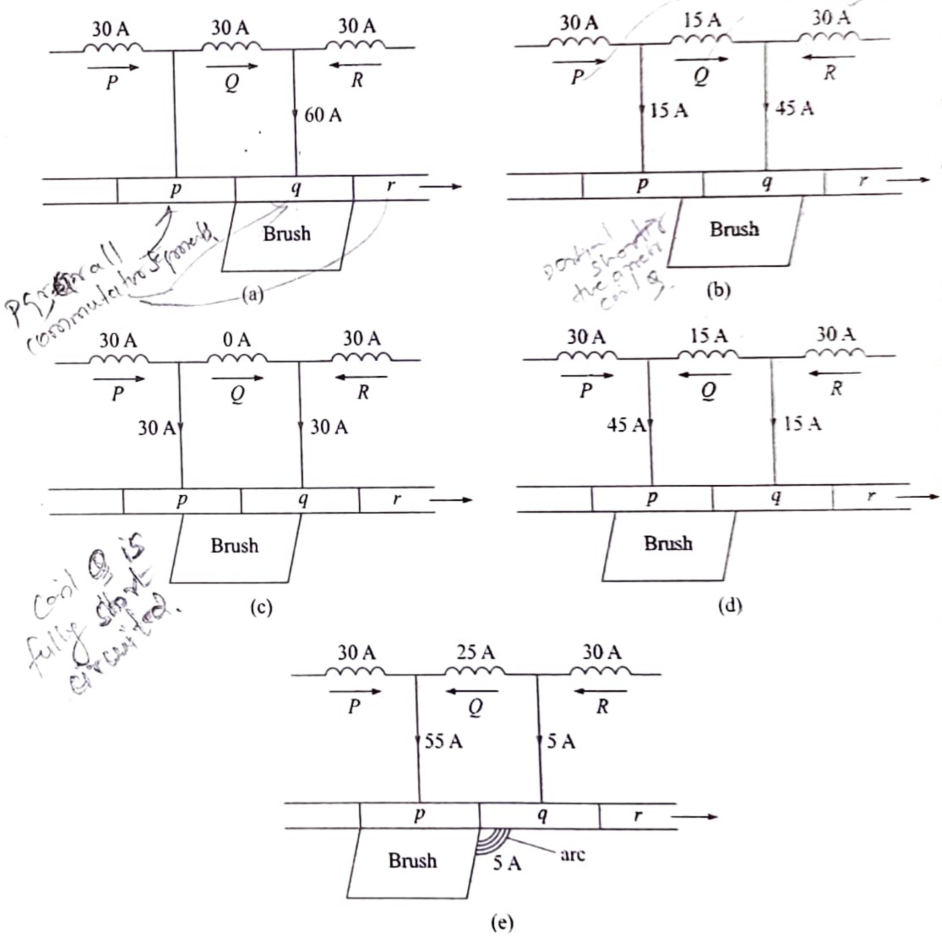
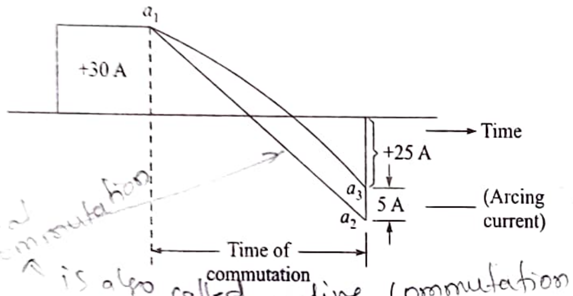


Figure 1.22 Different stages of commutation.

It is due to this reason, as shown in Figure 1.23, that the change in current in Q does not follow the straight line locus a_1a_2, rather it follows a_1a_3. As a result, this difference of 5 ampere current in the above example appears as an arc between the commutator and the brush as shown in Figure 1.22(e). This obviously has a deterimental effect on the life of the dc machine. To protect the dc machine from the above problem of commutation, an interpole is installed in each of the interpolar regions.



ideal commutation is also called - line commutation -> is that in which the current of the commutating coil changes linearly from +Ic to -Ic in the commutation period.

Figure 1.23 Variation of current in coil undergoing commutation from +30 A to –25 A.

Calculation of reactance voltage

We have seen that when an armature coil undergoes commutation, there is a change in current in the coil from a positive value I to a negative value $-I$. During this process of commutation, the reactance voltage is developed as explained above. The expression for the reactance voltage can be derived as follows:

Reactance voltage = Coefficient of self inductance × Rate of change of current

Time of commutation or short circuit

$$= \frac{w_b - w_m}{v_c}$$

where

w_b = width of the brush
w_m = width of the insulation between the commutator segments
v_c = peripheral velocity of the commutator.

Total change of current during commutation

$$= I - (-I) = 2I$$

$$\therefore \quad \text{Reactance voltage } = L \times \frac{2I}{\frac{w_b - w_m}{v_c}}$$

Methods of obtaining good commutation

There are two methods for achieving good commutation.

(a) Resistance commutation
(b) Voltage commutation.

In the case of resistance commutation, the resistance between the commutator segments and the brushes is made high by providing carbon brushes. This helps to increase the resistance in the circuit and accordingly the time constant L/R is reduced. Hence, the change in current during commutation becomes faster. The carbon brushes also help to reduce commutator wear. Moreover, a carbon brush can be easily replaced.

In the case of voltage commutation, the reactance voltage is neutralized by providing a suitable polarity dynamical voltage into the commutating coil. Narrow interpoles or commutating poles or compoles are provided for this purpose in the interpolar region (Figure 1.24). The interpoles have a winding in series with the armature.

Interpol

Interpole

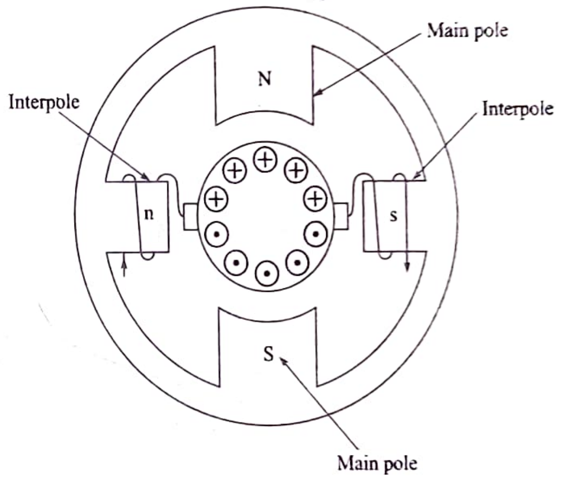


Figure 1.24 Position of interpoles in a two-pole dc machine.

The mathematical expression for obtaining the interpole amperetums required to cancel the armature reaction amperetums may be expressed as follows:

$$\text{Interpole amperetums required} = \text{Armature amperetums (peak)} + \frac{B_i}{\mu_0} L_{gi}$$

where

B_i = flux density in the interpolar air gap
L_{gi} = air-gap length between the interpole and the armature
μ_0 = permeability of the air

Armature reaction effect if not eliminated

If the effect of the armature reaction is ignored, then a large amount of distortion of field flux may occur along with the shifting of the magnetic neutral axis. Figure 1.25 describes the mmf distributions. When the brushes are placed on the geometric neutral axis, the armature mmf distribution, the flux density distribution for armature mmf only, the flux density distribution for main field only, and the resultant flux density distribution are all shown in Figure 1.25. The brushes will be lying on the geometric neutral axis. In the interpolar region, there is also a larger air gap and that is why, a larger dip in the quardrature axis is observed in respect of the flux density distribution for armature mmf.

EXAMPLE 1.5 A 6-pole dc generator has lap-wound armature. The generator has 720 conductors. The current carried by the load is 55 A at full-load condition. If the brush lead is 10°, determine the demagnetizing and cross-magnetizing ampereturns per pole.

Solution

Demagnetizing AT/pole

$$\mathrm{AT}_d = ZI_c \times \frac{\theta}{360}$$

$= \mathrm{ATa} \times \frac{2 \times \beta}{180}$

$= \frac{1}{2}\left(\frac{A\,op\,cond}{P}\right)$

$\left(\frac{720 \times 55}{6}\right)$

3300

$= 3300 \times \frac{2 \times 10}{(180)} \rightarrow 360$

$= 183.33$

where

Z = number of conductors

I_c = current per conductor

θ = brush lead

Now, $I_c = \frac{55}{6}$ ($\because$ number of parallel path in lap winding = number of poles = 6)

$$\therefore \qquad \mathrm{AT}_d = 720 \times \frac{55}{6} \times \frac{10}{360} = 183.33$$

Cross-magnetizing AT/pole

$\mathrm{ATa}\left(1 - \frac{2\beta}{180°}\right) \rightarrow 360$

$= 3300\left(1 - \frac{2 \times 10}{180}\right)$

$$= ZI_c\left(\frac{1}{2p} - \frac{\theta}{360}\right)$$

$$= 720 \times \frac{55}{6}\left(\frac{1}{2 \times 6} - \frac{10}{360}\right)$$

$= \mathrm{ATa} \times$

$\beta = \frac{P}{2}$ (in terms mech degree (or) angular degree mech).

$\beta = \frac{6}{2} \times 10°$

$\beta = 30$

$$= 6600(0.083 - 0.028) = 363$$

0.055

EXAMPLE 1.6 A 90 kW, 450 V, 4-pole dc shunt generator has a wave-wound armature of

(b) A straight line OT_1 is drawn which represents the field cricuit resistance 80 Ω.

$$\text{Critical speed} = N \times \frac{MQ}{PQ} = 850 \times \frac{99.7}{130} = 652 \text{ rpm}$$

(c) Emf induced due to residual flux = 11 V (i.e. when the field current is zero)

$$\therefore \quad \text{Flux per pole} = \frac{E \times a}{Z \times N \times p}$$

$$= \frac{11 \times 6}{580 \times \frac{850}{60} \times 6} = 0.0013 \text{ Wb}$$

1.8 CHARACTERISTICS OF DC GENERATOR

The dc generator has two types of characteristics:

(a) External characteristics

(b) Internal characteristics

The load characteristic of D.C generator at a particular speed its terminal voltage and load current (line current) is termed as external characteristic

The graphical representation of the terminal voltage vs. the armature or load current of a dc generator is termed the external characteristic of the dc generator. On the other hand, the graphical representation of the generated voltage vs. the load current is termed the internal characteristic of the dc machine.

1.8.1 Characteristics of Separately Excited DC Generator

Figure 1.32 shows the external characteristics of a separately excited dc generator. It shows the way in which the terminal voltage varies with the variation in load current from zero to full-load value. The speed of rotation and the excitation current are kept constant. The voltage drop from the no-load characteristic to the external characteristic depends on the following factors:

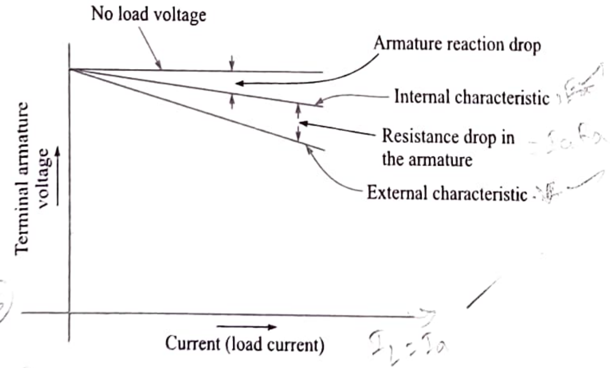


Figure 1.32 Characteristics of separately excited dc generator.

$V = E - I_a R_a$

(a) Armature reaction which produces a demagnetizing effect on the field flux.

(b) An internal voltage drop produced by the resistance of the armature winding.

1.8.2 Characteristics of DC Shunt Generator

Figure 1.33 describes the external characteristics of dc generator. In case of dc shunt generator, the field circuit is connected directly across the armature. With the increase in the load current, the voltage drops as a result of armature reaction and internal resistance of the armature circuit are increased and therefore the terminal voltage decreases. Hence the field current decreases. As a result, the terminal voltage drops further. At no-load the terminal voltage is the same as the generated voltage. The effects of armature reaction, armature circuit voltage drop, and decrease in field current are all shown in Figure 1.33 against the increase in load current.

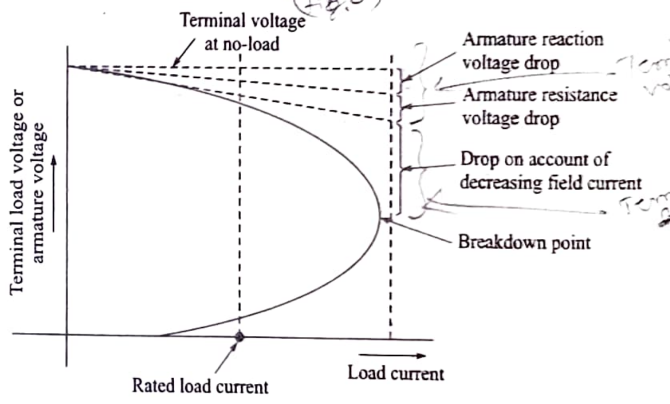


Figure 1.33 Characteristics of dc shunt generator.

The effects of both the armature reaction and the armature resistance voltage drop are shown as dashed straight lines in Figure 1.33. These dashed lines represent linear voltage decreases directly proportional to the increase in load current. The drop owing to decreased field current is represented by the curved line, since it depends on the degree of field flux saturation at that value of load. It can be seen from Figure 1.33 that the terminal voltage decreases with load current only to a small extent up to its rated load current value. Thus, the shunt generator produces a fairly constant output voltage with application of load. If further application of load continues, it causes the generator to reach a breakdown point where the armature reaction effect becomes so severe that the terminal voltage drops to a great extent and as a result, the generator cannot draw any larger load current. The load current, in fact, also drops.

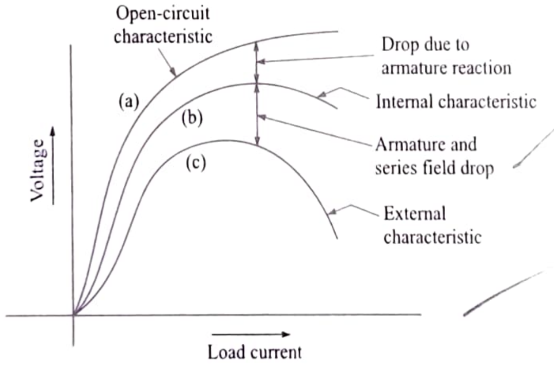
1.8.

(ix) From point u_4 a vertical line is drawn and that will intersect the line Nu_2 at u_5. The point u_5 will lie on the external characteristic. In a similar manner, the other points on the external characteristic can also be drawn.

19

1.8.4 Characteristics of DC Series Generator

~~Figure 1.36~~ describes the characteristics of the dc series generator. The curve (a) shows the open-circuit characteristic. Curve (c) is the external characteristic of the dc series generator. Curve (b) indicates the internal characteristic of the dc series generator. When the load current is zero, the generated and terminal armature voltages are same, both being due to residual magnetic field. Automatic build up of voltage takes place from the point when the load current flows through the series field winding producing additional flux aiding the residual flux. The load current produces two voltage drops as shown in Figure 1.36, thus limiting the voltage across the load. The generated voltage is also reduced by the effects of armature reaction. As a result of voltage drops tending to decrease the generated voltage and the magnetizing current tending to increase the generated voltage, a maximum voltage is produced where the buildup ceases. At this point the voltage drops of the series field and the armature as well as the voltage drop due to armature reaction exactly counterbalance the increased flux produced in the series field, and the terminal voltage therefore remains constant. Any further increase in load beyond the maximum voltage point produces a sharply drooping characteristic as illustrated in Figure 1.36. The reason being that the increased voltage drops and the increased armature reaction decrease the load voltage at a much faster than the increase in generated voltage taking place by the increased load current.



19(a)

Figure 1.36 Characteristics of dc series generator.

1.8.5 Characteristics of DC Compound Generator

Figure 1.37 describes the external characteristics of the dc compound generator. If the series excitation of the compound generator is made such that the terminal voltage on full-load is the

26(a)

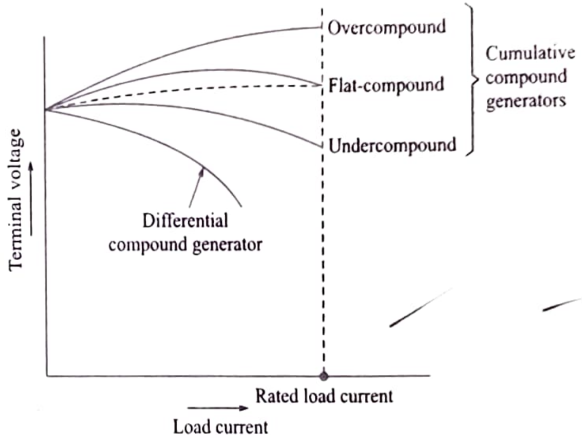


Figure 1.37 External characteristics of dc compound generator.

same as that of on no-load, then the dc generator is termed flat-compounded generator or level-compounded generator. If the series excitation becomes more prominent than that of the shunt field, then the terminal voltage enhances with the load. The generator is termed overcompounded. 30 When the prominency of the shunt field is more, then the terminal voltage is reduced with the increase in load and the compound dc generator is termed undercompounded dc generator. In case of the differential compound generator, the series field is connected such that its field opposes the shunt field. For the differential compound generator, the terminal voltage drops very quickly with the increase in load current.

EX

1.9 PARALLEL OPERATION OF DC GENERATORS

When the load is more than the rating of a single generator, then the generators are connected in parallel to meet the demand of load. For maintaining a reliable system with continuity of service the parallel operation from that angle is also desirable. Over and above, for most efficient operation, the parallel operation of generators is always essential.

1.9.1 Conditions Necessary for Parallel Operation of DC Generators

For parallel operation of dc generators, the following conditions need to be satisfied:

(a) The terminal voltage of each generator must be the same.
(b) The polarities of the generators must be same.
(c) The prime movers driving the generators must have similar and stable characteristics from the point of view of rotation.
(d) The change in voltage with respect to the change in load must be of the same order for all generators.

1.9.2 Procedure for Connecting DC Shunt Generators in Parallel

The generator G_1 is first connected to the load as shown in Figure 1.39. The generator G_2 is now to be connected in parallel with the generator G_1.

The procedure used to parallel one generator to another is as follows:

(a) Generator G_2 is speeded up by the prime mover until its rated voltage builds up. Then the switch 'S' is closed.
(b) The excitation of the generator G_2 is adjusted until the reading of the voltmeter connected across switch S_2 becomes zero.
(c) The switch S_2 is now closed.

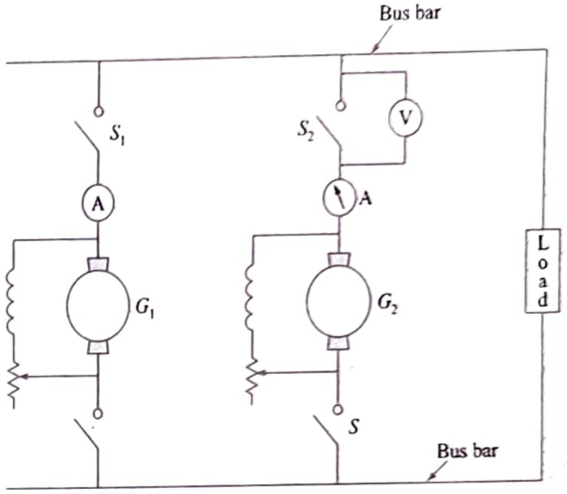


Figure 1.39 Parallel operation of two generators.

(d) Now the two generators are in the floating condition since the voltage of both are the same. The shifting of the load from the generator G_1 is then made to generator G_2. The field rheostat resistance of generator G_1 is increased whereas the field rheostat resistance of generator G_2 is decreased.
(e) Gradually the sharing of load by the two generators will occur.

Figure 1.40 describes the characteristics of two shunt generators working in parallel.

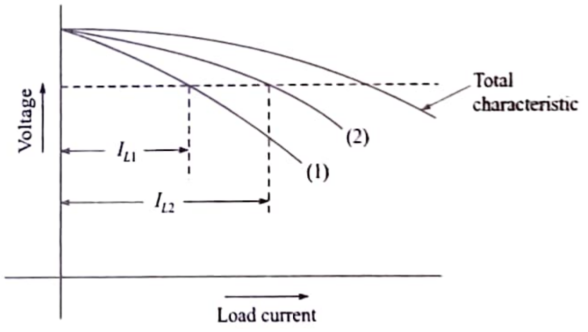


Figure 1.40 Characteristics of shunt generators in parallel.

1.9.3 Parallel Operation of Compound Generators

The main problem of parallel operation of compound generators is instability. Suppose, one of the generators begins to deliver more load current than what it was actually delivering. The field of this generator will be strengthened, causing an increase in its generated voltage. Since

$E = V + Ir$, for a constant terminal voltage V, the generator will start to take more load current. When the load is fixed, the other generator will then take less load. So its field will be weakened and the load current will be reduced further. If this process continues, one generator will take all the load and the other generator will not deliver any load current. Finally, the unloaded generator will acts as a motor to the other heavily loaded generator. Thus an instability will occur in the system. This condition should not be allowed. To avoid this situation, an equalizer is connected to the armature side of the series field on the side of the same polarity for each generator, as shown in Figure 1.41. Equalizer is the low resistance connection. Now any variation in the induced emf of one generator will produce a circulating current only between the two generators and the equalizer. The series fields will not be affecfed by this variation.

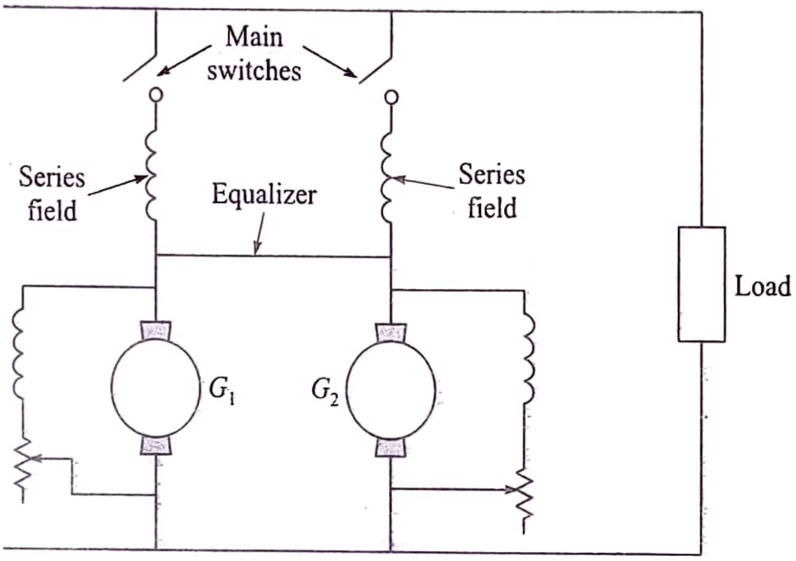


Figure 1.41 Compound generators in parallel.

1. What are the methods of speed control in DC motor? (Apr/may 2010)

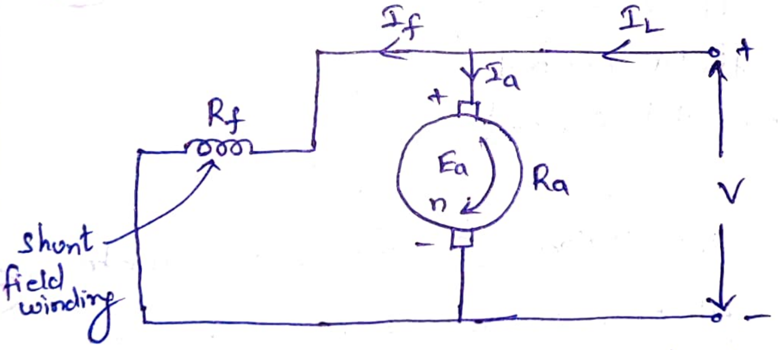
(i) Shunt motor speed control

- Field control of DC shunt motor
- Armature resistance control of dc shunt motor
- Series parallel control of DC shunt motor
- Ward Leonard method of speed control

(ii) Series motor speed control

- Diverter field control
- Tapped field control
- Series - parallel control

2. Draw the circuit model of DC shunt motor. Apr/may 2011



circuit model of dc shunt motor

5. why a fly wheel is used with dc series motor? (may/Jun

Fly wheel It is act as a load and its avoid the dangerous speed at the time of starting (due to high starting torque)

---

6. why DC series motor is not suitable for belt driven loads? (may/Jun 2012)

The DC series motor are the variable speed, high starting torque motor for which they are not suitable for belt driven loads.

---

7. state the methods of speed control in dc series motor. (Nov/Dec 2012)

(i) Diverter field control
(ii) Tapped field control
(iii) series parallel control

---

8. write down the applications of d.c series motor. (may/Jun 2013)

(i) speed regulation of a d.c series motor can be varied widely.
(ii) For drives requiring a very high starting torque, such as Hoists, cranes, bridges, battery-powered vechicles and traction type loads.

---

9. write the emf equation of a d.c machine. (Nov/Dec-2013)

$$\text{Induced emf, } E_a = \frac{\phi z n}{60} \times \left(\frac{P}{A}\right) \text{ volts.}$$

where

$A = 2$ for simplex wave winding
$A = P$ for simplex lap winding
ϕ = flux per pole in webers
Z = Total no of Armature conductors
n = Armature rotations speed in revolutions per minute (rpm)

U-III

1.11 DIFFERENT TYPES OF DC MOTORS

DC motors like the dc generators are also of different types.

(a) Shunt wound dc motor
(b) Series wound dc motor
(c) Compound wound dc motor.

The compound wound dc motor is also of two types: (i) Cumulative compound wound dc motor and (ii) Differential compound wound dc motor.

In case of the cumulative compound wound dc motor, the field windings are connected in such a way that the fluxes developed both in the shunt and series windings are in the same direction. In case of the differential compound dc motor, the scenario is just the opposite.

1.12 CHARACTERISTICS OF DC MOTORS

The characteristics of dc motors mainly express the speed–torque characteristic of the motor. Let us first of all consider the case of the dc shunt motor.

DC shunt motor

We know the following relations:

$$E = \phi NZ \frac{p}{a} = K_1 \phi N$$

$$V = E + Ir$$

$$T = K_2 \phi I$$

where

E = back emf
V = supply voltage
I = armature current
r = armature resistance
K_1, K_2 = constants
ϕ = flux per pole.

$$\therefore \quad N = \frac{E}{K_1\phi} = \frac{V - Ir}{K_1\phi}$$

$$= \frac{V - \frac{T}{K_2\phi} . r}{K_1\phi}$$

or

$$N = \frac{V}{K_1\phi} - \frac{Tr}{K_1K_2\phi^2}$$

From the above expression it is clear that the speed–torque characteristic should be a straight line if the flux per pole remains constant. But due to armature reaction effect, the above condition never happens. Figure 1.44(c) describes the drooping characteristic of speed with respect to the torque. Both the speed and torque vary with respect to the armature current as shown in Figure 1.44(a) and Figure 1.44(b) respectively. Due to the effect of armature reaction, the flux reduces and the speed increases with the increase in armature current. Also, due to the effect of armature reaction, the flux reduces and the torque decreases with the increase in armature current, since for a particular power, the torque will reduce with the increase in speed.

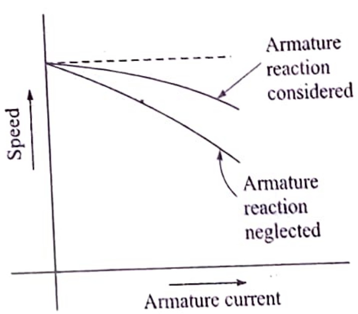


Figure 1.44(a) Speed vs. armature current of dc shunt motor.

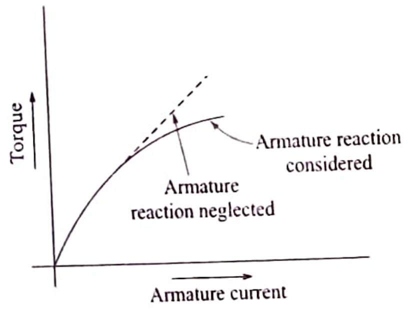


Figure 1.44(b) Torque vs. armature current of dc shunt motor.

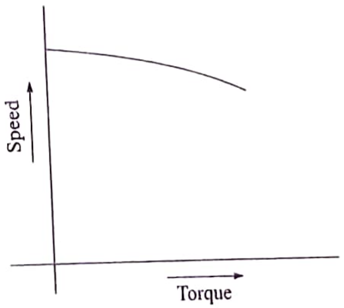


Figure 1.44(c) Speed vs. torque of the dc shunt motor.

DC series motor

As regards the dc series motor the following mathematical relations are found:

$$\phi = K_3 I$$

$$E = K_1 \phi N$$

$$V = E + I(r + r_f)$$

$$T = K_2 \phi I$$

$$\therefore \quad N = \frac{E}{K_1\phi} = \frac{V - I(r + r_f)}{K_1\phi}$$

$$= \frac{V - I(r + r_f)}{K_1K_3I}$$

$$= \frac{1}{K_1K_3}\left[\frac{V}{I} - (r + r_f)\right]$$

Also,

$$T = K_2\phi I = K_2(K_3I)I = K_2K_3I^2$$

$\therefore$

$$N = \frac{1}{K_1K_3}\left[\frac{V\sqrt{K_2K_3}}{\sqrt{T}} - (r + r_f)\right]$$

Here, $N = \frac{1}{K_1K_3}\left[\frac{V}{I} - (r + r_f)\right]$ is a curve of shifted rectangular hyperbola and $T = K_2K_3I^2$ is a parabola.

A slight difference is observed in the characteristics being obtained from the mathematical result and those from the practical result as shown in Figures 1.45(a), (b) and (c) because saturation and armature reaction demagnetization make the flux/pole to increase at a rate slower than what is being considered in the mathematical expression.

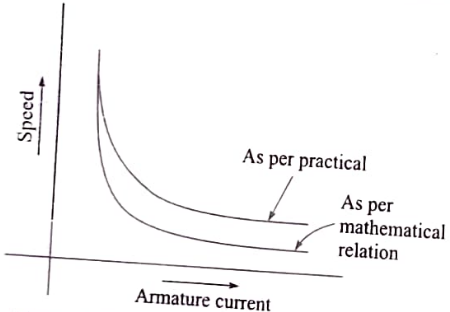


Figure 1.45(a) Speed vs. armature current of dc series motor.

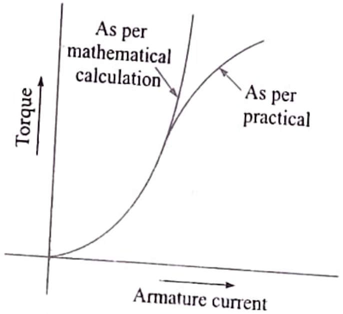


Figure 1.45(b) Torque vs. armature current of dc series motor.

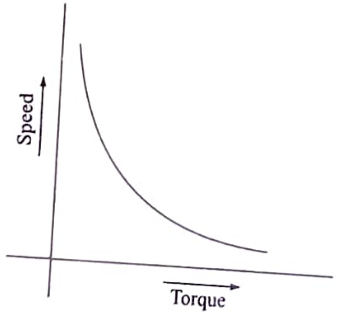


Figure 1.45(c) Speed vs. torque for the dc series motor.

The most important point of the characteristic of the dc series motor is that at no-load, the motor current and the flux/pole tend to zero and as a result, the motor speed tends to increase to infinity. This is a dangerous condition and the centrifugal force experienced may damage the armature severely. Therefore, the dc series motor should never be run at no-load even by mistake. Near the saturation region, ϕ does not remain proportional to the armature current I and therefore the relation $T = K_2K_3I^2$ tends to $T \propto I$ and as a result as per the mathematical relation as the armature current rises the parabolic curve tends towards the straight line curve. This is clear in the mathematical related curve of Figure 1.45(b).

DC compound motor

For the case of the dc compound motor, Figures 1.46(a), 1.46(b) and 1.46(c) describe the speed vs. current, torque vs. current and speed–torque characteristics of the motor, respectively.

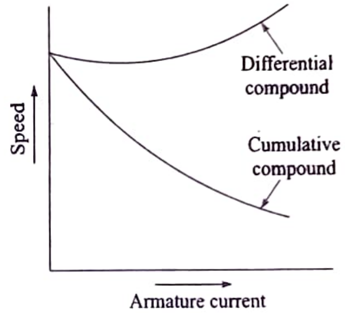


Figure 1.46(a) Speed vs. armature current characteristics of dc compound motor.

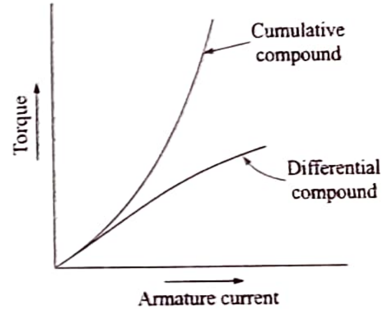


Figure 1.46(b) Torque vs. armature current characteristics of dc compound motor.

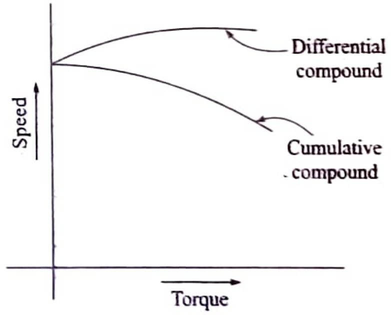


Figure 1.46(c) Speed vs. torque characteristics of dc compound motor.

1.14 SPEED CONTROL OF DC MOTORS

The dc motor plays a very important role in the control of industrial drives. A wide range of speeds of drives can be obtained using dc motors.

As we know,

$$V = E + Ir$$

or $$V = K\phi N + Ir$$

or $$N = \frac{V - Ir}{K\phi}$$

or $$N \approx \frac{V}{K\phi}$$

if the Ir drop is neglected. Therefore the speed variation depends on two factors, i.e. V and ϕ. So the terminal voltage and field excitation help to vary the speed of dc motor.

Let us now analyse the speed control of various types of dc motors.

1.14.1 Shunt Motor Speed Control

In case of the dc shunt motor, the following equations have already been discussed.

$$V = E + ir$$

$$E = K_1\phi N$$

$$T = K_2\phi I$$

$$\therefore \quad N = \frac{E}{K_1\phi} = \frac{V - Ir}{K_1\phi} = \frac{V}{K_1\phi} - \frac{Ir}{K_1\phi}$$

$$= \frac{V}{K_1\phi} - \frac{\frac{T}{K_2\phi}r}{K_1\phi}$$

or $$N = \frac{V}{K_1\phi} - \frac{Tr}{K_1K_2\phi^2}$$

may be called the basic speed equation of the dc shunt motor.

Field control of dc shunt motor

Figure 1.51 describes the speed control of dc shunt motor by variation in field excitation. Figure 1.52 shows the speed vs. torque characteristics. As the field current is increased, the speed–torque characteristic moves down. The speed–torque characteristics shown in Figure 1.52 are with three values of field current, I_{f_1}, I_{f_2} and I_{f_3}, where $I_{f_1} < I_{f_2} < I_{f_3}$.

For a constant field current, due to demagnetization of field on account of armature reaction, the actual speed is found a little higher for the same torque produced. So, two curves are shown in Figure 1.52 for each value of field current.

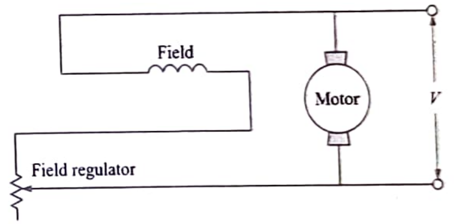


Figure 1.51 Field control of dc shunt motor.

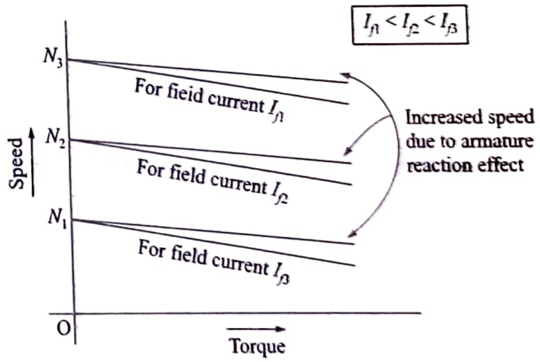


Figure 1.52 Speed control of dc shunt motor by variation in field excitation.

Armature resistance control of dc shunt motor

As the armature circuit resistance of the motor is increased (Figure 1.53(a)) for a particular torque the back emf decreases. Hence for a constant field flux, the speed decreases.

Figure 1.53(b) exhibits the fall of speed-torque curve with the increase in the armature circuit resistance R.

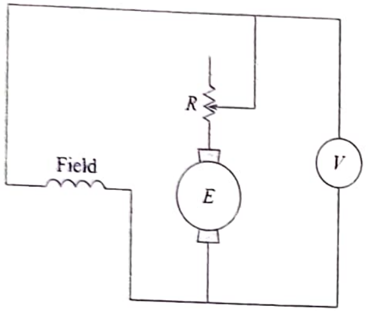


Figure 1.53(a) Armature resistance control of dc shunt motor.

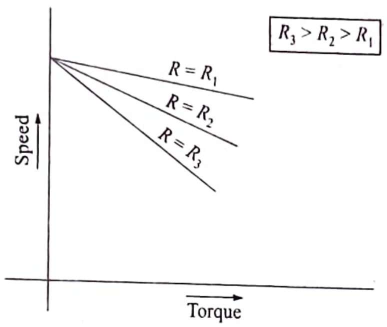


Figure 1.53(b) Speed control of dc shunt motor by varying the atmature circuit resistance.

The main problems of this type of speed control are the following:

(i) Speed below the rated value is possible.

(ii) Speed variation cannot be made wide because of the reduction in the efficiency of the motor.

(iii) Speed regulation is poor.

At no-load condition,

$$V \approx K\phi N$$

Neglecting armature resistance at armature current I,

$$V - IR = K\phi N_1$$

$$\therefore \quad N - N_1 = \frac{V}{K\phi} - \frac{V - IR}{K\phi}$$

$$\therefore \quad \text{Speed regulation} = \frac{N - N_1}{N}$$

$$= \frac{\frac{V}{K\phi} - \frac{V - IR}{K\phi}}{\frac{V}{K\phi}}$$

$$= \frac{\frac{V - V + IR}{K\phi}}{\frac{V}{K\phi}} = \frac{IR}{V}$$

So, for a fixed value of R, the speed variation totally depends on the load.

Series-parallel control of dc shunt motor

In the series-parallel method of speed control of dc shunt motor, two identical shunt motors are connected mechanically to a common load. Here two speeds can be obtained. In one case, the armatures are connected in series as shown in Figure 1.54(a). In the other case, the armatures are connected in parallel as shown in Figure 1.54(b).

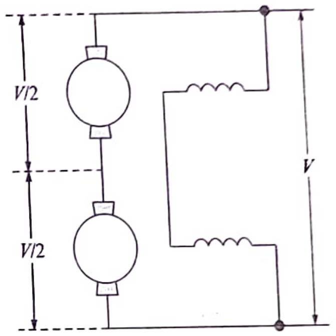


Figure 1.54(a) Armatures in series.

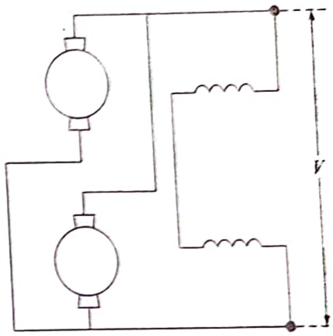


Figure 1.54(b) Armatures in parallel.

When the armatures are connected in series, the voltage supply to each armature is reduced to half and when the parallel connection is made, each motor will be supplied the full voltage. Thus the speed of the combination can be arranged to be either the half speed or the full-rated speed.

Ward Leonard method of speed control

Combined armature and field control of dc motor can be performed by a famous method which is called the Ward Leonard method of speed control (Figure 1.55).

The Ward Leonard method provides a wide range of speed control. When a three-phase ac supply is provided to an ac induction motor, it rotates along with the exciter and the dc generator, all three being mounted on a common shaft as shown in Figure 1.55.

The exciter supplies power to the variable resistor and the variable voltage is applied by the potentiometric arrangement through the resistor to the field of the generator. Thus the voltage across the field of the dc generator is varied. Hence the voltage of the generator varies. This variable voltage is applied to the armature of the dc motor whose speed is to be controlled. At the same time the exciter voltage is applied to the field of the dc motor through a variable rheostat. Hence both the voltage across the armature as well as the voltage across the field of the dc motor are varied widely so that speed control over a wide range can be obtained. The main advantage of the Ward Leonard method is the realization of the wide range of speed control of the dc motor. But the main disadvantage is that it is a very costly arrangement because a large number of rotating machinaries are being used for the speed control of the dc motor.

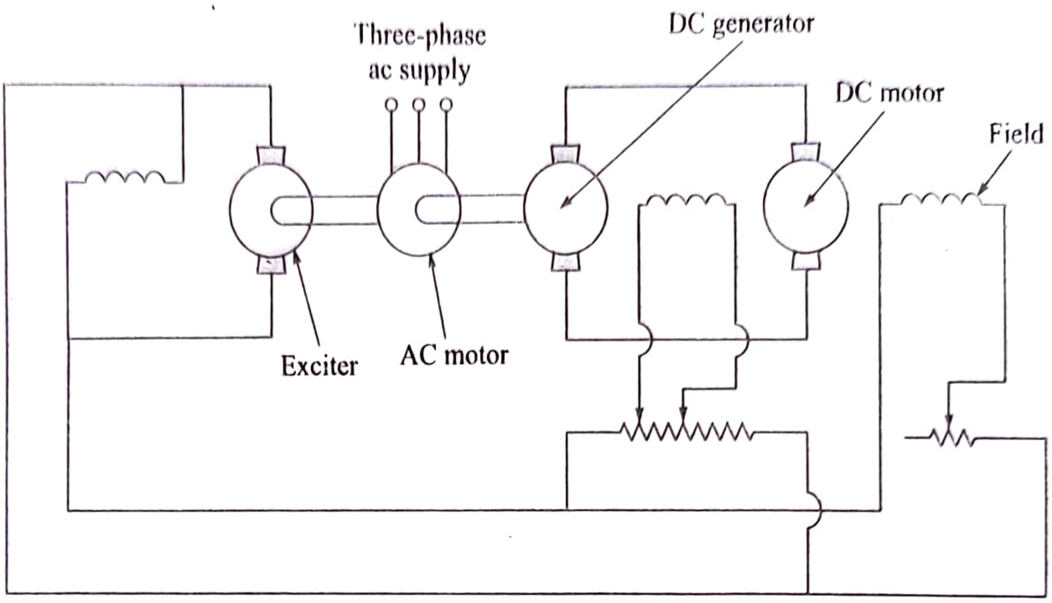


Figure 1.55 Ward Leonard speed control of dc shunt-motor.

1.14.2 Series Motor Speed Control

The series motor speed control like the shunt motor speed control can also be made by varying the field excitation as well as exercising the armature-circuit resistance control. The methods used are as follows:

(a) Diverter field control
(b) Tapped field control
(c) Series-parallel control.

Diverter field control

Figure 1.56 describes the diverter resistor control circuit used for speed control of dc series motor. A variable diverter resistance r_d is connected across the series field having resistance r_{se}.

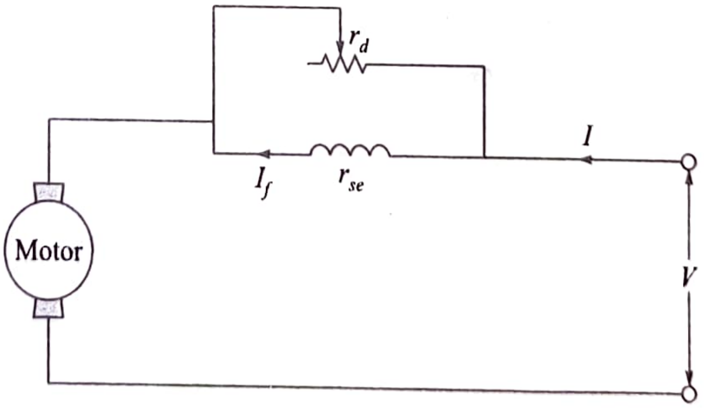


Figure 1.56 DC series motor speed control using diverter field control.

1.13 STARTING OF DC MOTOR

At the instant when a voltage is applied to a dc motor to start it, it is at the stationary condition. So, there is no back emf generated in the armature of the motor since the speed is zero. Therefore, only the armature resistance in the circuit exists for limiting the in-rush of the current from the mains supply. So, if the rated voltage is applied across the motor, a large amount of current will flow through the motor and the motor will get damaged. Hence, at the time of starting some arrangement needs to be made so that an enormous amount of current does not flow through the motor armature. Gradually, as the motor speeds up, the starting arrangement is withdrawn. Such a device which is provided with the dc motor at the time of starting is called the *starter*. Before the advent of power electronics, a manual starter was the only device used for starting of a dc motor.

1.13.1 Manual Starters

The manual starters are usually of two types:

(a) Three-point starter
(b) Four-point starter

Figures 1.47(a), 1.47(b), 1.48(a) and 1.48(b) are the starters of dc motor. Figure 1.47(a) and Figure 1.47(b) are the three-point starters. They are more or less of similar type. The only difference is that in Figure 1.47(b), a brass arc has been provided for making the operation more effective. Figure 1.48(a) and Figure 1.48(b) are the four-point starters. In Figure 1.48(b), the brass arc is additionally provided for the same purpose as mentioned above. The major difference between the three-point starter and the four-point starter is that the four-point starter possesses four terminals, whereas the three-point starter possesses three terminals.

Let us take the case of the three-point starter as shown in Figure 1.47(b). The step-by-step principle of operation of the starter is described as follows:

1. The dc supply is switched on.
2. The starting arm is slowly moved from its OFF position to the studs 1, 2, 3, 4, 5. As a result, first of all the full starting resistance at stud 1 is inserted in the armature circuit. Then, gradually, the step resistances shown are cut off from the armature circuit as the starting arm is moved from stud 1 through stud 5. Gradually, at the same time the step resistances go to the field circuit of the dc motor through the brass arc. In Figure 1.47(a), of course, there is no brass arc. The resistances go to the field circuit through the wire connection.

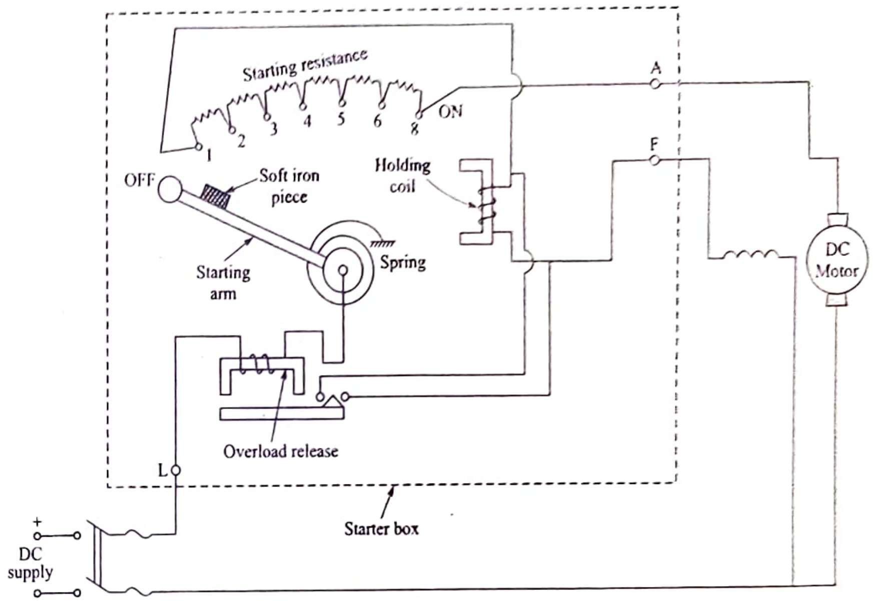


Figure 1.47(a) Three-point starter.

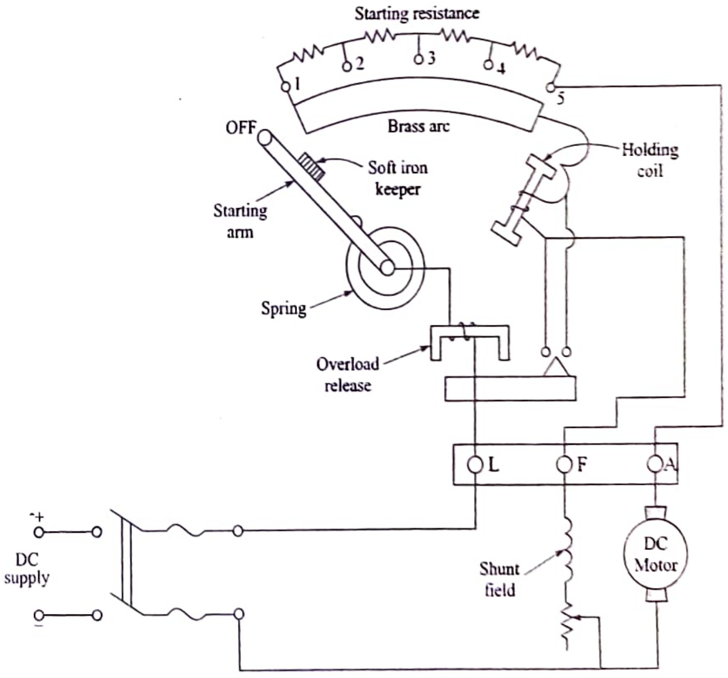


Figure 1.47(b) Three-point starter with brass arc.

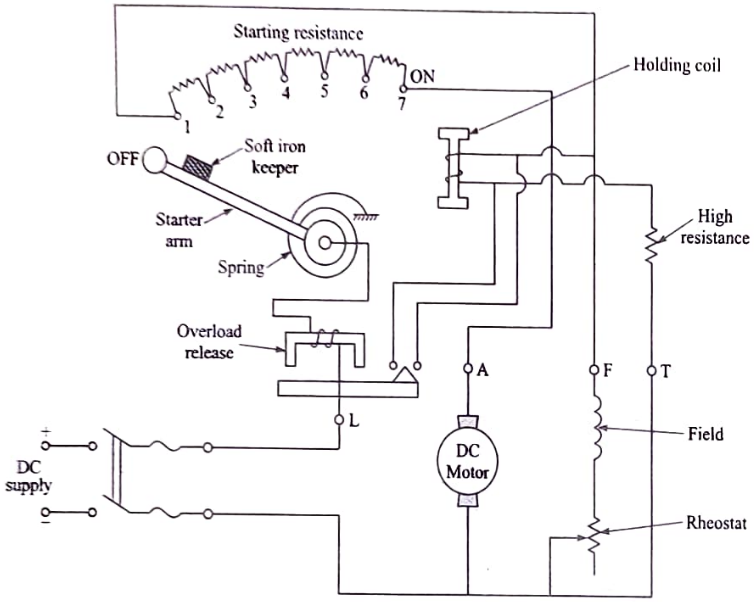


Figure 1.48(a) Four-point starter.

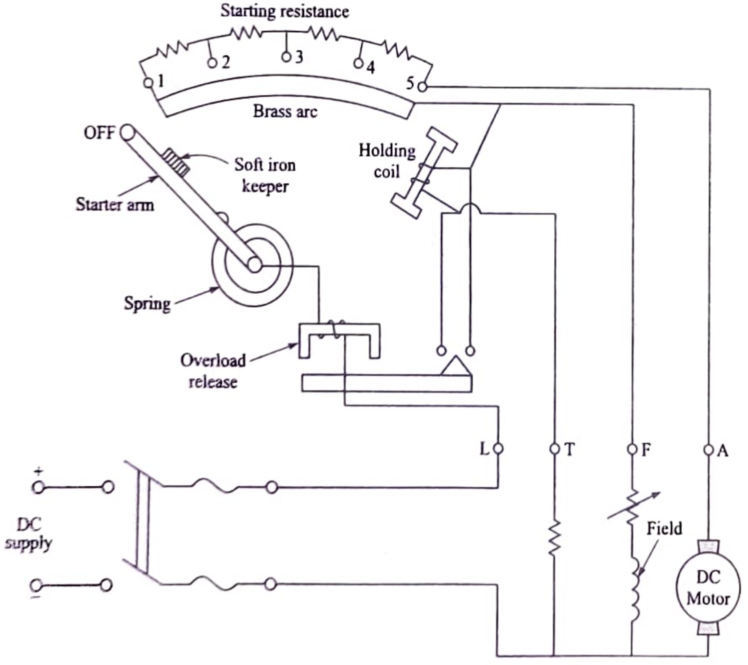


Figure 1.48(b) Four-point starter with brass arc.

3. When the starting arm reaches to the last stud point (it may be 5 or 6 or 7 etc. as per the design) it will remain at that position because the attached soft iron keeper of starting arm remains attracted to the holding coil magnetic strength.
4. If the motor is switched off, then the holding will be de-energised and the starting arm will come back to its original OFF position.
5. If the load current becomes large, the overload release coil will lift the armature and the holding coil is shorted. Thus the holding coil is de-energised. Automatically, the starting arm then returns to the OFF position.

In case of the four-point starter, the holding coil circuit is not connected to the field circuit of the motor. Rather an extra terminal is provided with a high resistance. This makes the system more reliable because if by chance the current flowing through the field circuit is not sufficiently high to magnetize the holding coil, then the soft iron keeper will no longer remain attracted and the starting arm will not lie at the final stud position.

1.16 TESTING OF DC MACHINES

The testing of dc machine is very essential so that we can accurately calculate the losses of machine and find out its efficiency. The efficiency of the dc machine is the ratio of output and input. The efficiencies of generator and motor can be expressed as follows:

$$\text{Efficiency of generator} = \frac{VI}{VI + \text{losses}}$$

$$\text{Efficiency of motor} = \frac{VI - \text{losses}}{VI}$$

In case of generator, V is the terminal voltage and I is the load current. In case of motor, V is the supply voltage and I is the supply current. The main two tests of dc machines are:

(1) Swinburne's test
(2) Hopkinson's test.

1.16.1 Swinburne's Test

Swinburne's test is the no-load test of dc machine. Therefore, this test cannot be performed on the dc series motor.

Figure 1.72 descrbes the circuit diagram of the Swinburne's test. The machine, whether it is a motor or a generator, is first of all run at no-load at the rated speed and at the rated terminal voltage. Then the field current is adjusted to its rated value. The no-load loss is calculated. A series resistor is inserted in the armature circuit of the motor so that it runs exactly at the rated speed. Now,

$$VI_a = P_0 + P_f + I_a^2 r_a$$

where

P_0 = iron loss
P_f = windage and friction loss
r_a = armature resistance

$$\therefore \quad P_0 + P_f = VI_a - I_a^2 r_a$$

Again the shunt field loss,

$$P_{sh} = I_f^2 r_f = VI_f$$

where r_f = field resistance.

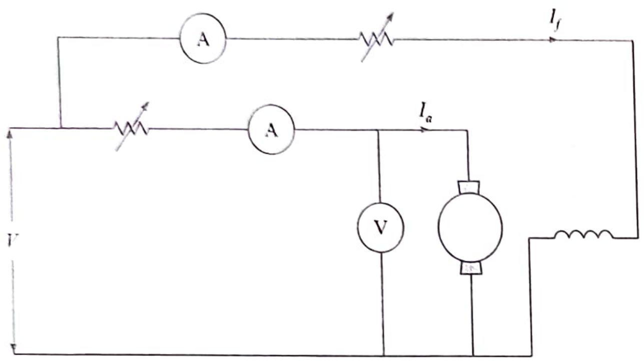


Figure 1.72 Swinburne's test.

The stray load loss can be considered 1% of the rated output at full-load or sometimes it is neglected.

Total loss, $P_T = (VI_a - I_a^2 r_a) + I_f^2 r_f$

If the dc machine acts as a generator, then the efficiency of the same will be

$$\eta_g = \frac{VI_L}{VI_L + P_T}$$

where I_L is the load current of the generator.

If the dc machine acts as a motor, then the efficiency will be

$$\eta_m = \frac{VI_L - P_T}{VI_L}$$

The Swinburne's test is not at all an accurate test for determining the efficiency of dc machines. The reasons are the following:

(a) The stray load loss cannot be calculated by this method. Therefore, the efficiency will not be accurate even if we assume stray load loss as some percentage of the rated output at full-load.

(b) Since the resistances r_a and r_f are measured at normal temperature, we are not able to get the actual loss at the real loading condition.

Besides the above, by the above test, we cannot get the actual scenario of commutation when the machine will be really loaded at the rated condition. Over and above, we cannot have an idea of steady temperature rise of the machine.

Now the question arises, what do we have to do? If it is a big machine, it is not advisable to fully load the machine, and thus make testing of the machine prohibitively expensive. That is why, some other method is needed where we can load the machine ficticiously at the rated full-load condition without really applying the full load. That is possible by the Hopkinson's test.

1.16.2 Hopkinson's Test

In the Hopkinson's test, we require two identical dc machines. This is a regenerative test. Both the dc machines are mechanically and electrically coupled, and are tested simultaneously.

One of the machines will act as dc motor and the other as dc generator. The motor will rotate the generator and the generator will supply power to the motor. Now the queston arises, if both machines help each other, then which one will supply the losses. The answer is external dc supply, that means, the input of the external supply will provide the power losses of both the machines. Now the voltage across switch S should be zero and only then we can declare that similar polarities of the machines are connected. So, when the switch S is closed both the dc machines are in perfect parallel connection and there is no chance of the presence of circulating currents between the two machines.

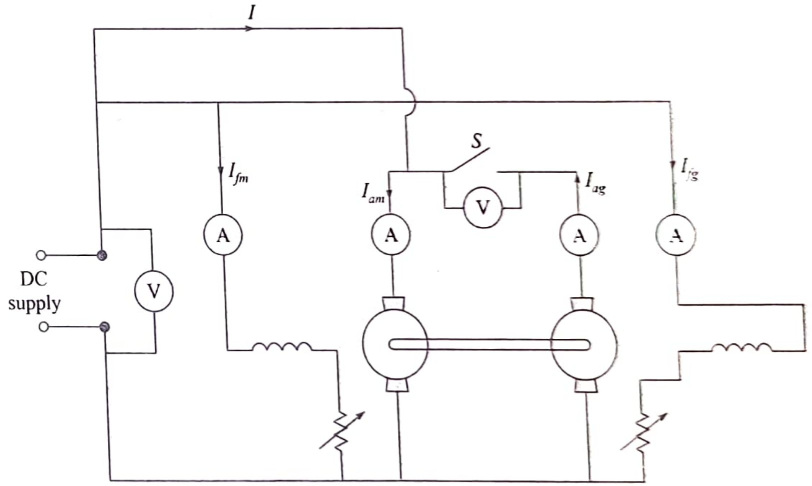


Figure 1.73 Hopkinson's test.

Since in case of generator the induced emf minus the armature drop is the terminal voltage and in case of motor the induced emf plus the armature drop is the terminal voltage, the excitation of the generator will be more than the excitation of the motor to maintain the terminal voltages equal to one another. Obviously, the no-load iron loss and stray loss will not be equal for both the machines even though the machines are identical in all respects. Stray load loss is a very important factor for determining the efficiency of dc machine. If we ignrore this, we cannot determine the actual efficiency of dc machine properly due to the following reasons. Stray load loss is the additional copper loss which occurs in the conductors an account of non-uniform distribution of alternating currents. This increases the effective resistance of conductors and that is nothing but the skin effect.

When the conductors carry load current, the teeth of the core get saturated and as a result, more flux passes down the slots through the copper conductors and thus setting up the eddy current losses in them. Even in winding overhang, the eddy current losses occur. Due to the flux distortion, the net increase in the core loss occurs and the extra core loss is nothing but the core stray load loss.

Mathematical analysis of Hopkinson's test

Now, in Hopkinson's test,

$$\text{Motor output} = \text{generator input}$$

i.e.

$$VI_m \eta_m = \frac{VI_g}{\eta_g}$$

where

I_m = motor current
I_g = generator current
η_m = motor efficiency
η_g = generator efficiency.

∴

$$\eta_m \eta_g = \frac{I_g}{I_m}$$

Then

$$\eta^2 = \frac{I_g}{I_m}$$

or

$$\eta = \sqrt{\frac{I_g}{I_m}}$$

$$I_m = I_g + \text{external supply current}$$

The motor current becomes equal to the generator current plus the current coming from the supply mains.

Hence, the efficiency of dc machine becomes

$$\sqrt{\frac{I_g}{I_m}}$$

Detailed calculation of efficiency of dc machine

Input to the armature circuit = VI

Total stray loss, $W = VI - I_{am}^2 r_{am} - I_{ag}^2 r_{ag}$

where r_{ag} and r_{am} are the armature resistances respectively.

(The stray loss includes windage and friction loss, no-load iron loss, stray load loss)

Motor field copper loss = VI_{fm}

Generator field copper loss = VI_{fg}

The stray loss is assumed half of W for each machine although it is not absolutely correct since the iron loss and stray load loss will not be the same for motor and generator because of the variation in the excitation of both of machines.

Efficiency of the motor

$$= \frac{VI_{am} + VI_{fm} - \frac{W}{2} - VI_{fm}}{VI_{am} + VI_{fm}}$$

$$= \frac{VI_{am} - \frac{W}{2}}{VI_{am} + VI_{fm}}$$

Efficiency of the generator

$$= \frac{VI_{ag}}{VI_{ag} + \frac{W}{2}} + VI_{fg}$$

TRANSFORMERS - unit-II

1. Define Transformer.

A Transformer is a static device comprising coils coupled through a magnetic medium connecting two ports at different voltage levels (in general) in an electric system allowing the interchange of electrical energy between the ports in either direction via the magnetic field.

---

2. Give the principle of transformers. (Apr/may 2010)

Transformer operates on the principle of mutual induction between inductively coupled coils. when A.C source is connected to one coil flux is produced in the core which links both the coils. As per faraday's laws of electromagnetic induction EMF is induced in the secondary coil also. if the external circuit is closed power is supplied.

$$E_1 = N_1 \frac{d\phi}{dt}$$

---

3. Define step-up transformer and step-down transformer:

If the secondary voltage is grater than the primary value, the transformer is called a step-up transformer; if it is less, it is known as a step-down transformer.

$N1 > N2 \rightarrow$ step down T/f

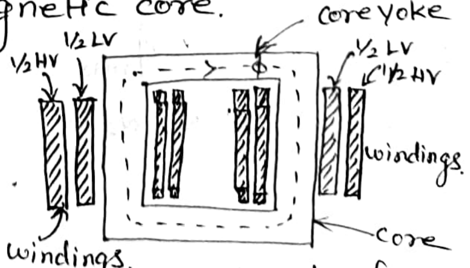
$N1 < N2 \rightarrow$ step up T/f

---

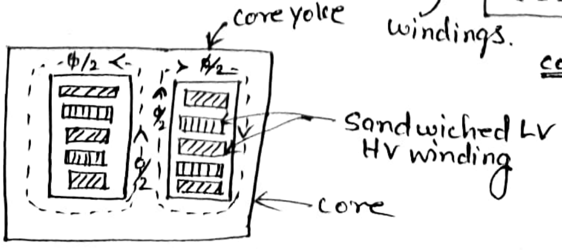
4. Mention the difference between core and shell type transformers. (may/Jun 2011) (NOV/DEC 2012) (NOV/DEC 2008)

* In core-type transformer the windings are wound around the two legs of a rectangular magnetic core.

* In shell-type transformer, the windings are wound on the central leg of a three-legged core.



core-type transformer.



Shell-type transformer

⑤ What are most important task's performed by Transform

(i) changing voltage and current levels in electric power sy

(ii) Matching source and load impedances for maximum power transfer in electric and control circuitry.

(iii) Electrical isolation [isolating one circuit from another (or) isolating dc while maintaining a.c continuity between two circuits]

---

6. Why transformer rating is expressed in terms of KVA? (Nov/Dec 2006) (Apr/May 2008)

Copper loss of a transformer depends on current and iron loss on voltage. Hence total losses depend on volt-ampere and not on the power factor. Thats why the rating of transformers is in KVA.

---

7. What are the losses in a transformer? (May/Jun 2013) (Apr/may 2008)
How will you minimise them?

(i) core loss
(ii) copper loss (I^2R- loss)
(iii) Load (stray)-loss
(iv) Diectric-loss.

→ Iron losses includes Hysteresis loss and Eddy current loss. Hysteresis loss can be minimized by choosing a core having small area of B-H loop curve and Eddy current losses can be minimized by laminating the core.

→ copper loss is minimized by reducing the leakage flux which is linked with both primary and secondary winding.

---

8. Write the E.M.F equation of two winding transformer.

Emf induced in primary coil $E_1 = 4.44 f N_1 \phi_{max}$ volts.

Emf induced in secondary coil $E_2 = 4.44 f N_2 \phi_{max}$ volts.

ϕ_{max} → Maximum value of core flux in webers.

N_1, N_2 → Number of primary and secondary turns.

9. List out the properties of ideal transformer.

(i) The primary and secondary windings have zero resistance. [It means that there is no ohmic power loss and no resistive voltage drop in the ideal transformer.]

(ii) There is no leakage flux so that all the flux is confined to the core and links both the windings.

(iii) The core-loss (hysteresis as well as eddy-current loss) is consider zero.

(iv) The core has infinite permeability so that zero magnetiz current is needed to establish the requiste amount of flux $\left(\phi_{max} = \frac{E_1 (=V_1)}{4.44 f N_1}\right)$ in the core.

---

10. What are the conditions for parallel operation of transformers? (Apr/may 2010)

(i) secondary voltages must be same

(ii) polarity must be same

(iii) phase sequence must be same

(iv) percentage impedance must be Same.

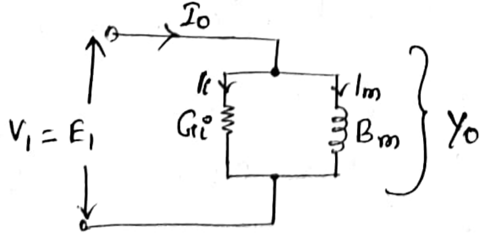
(v) $\frac{X}{R} = \frac{\text{Equivalent leakage Reactance}}{\text{Equivalent Resistance}}$, ratio must be same

(vi) frequency rating must be same.

---

11. which equivalent circuit parameters can be determined from the open circuit test on a transformer? (Apr/may 2011)

The OC test yields the values of core-loss and parameters of the shunt branch of the equivalent circuit.



$Y_o = G_i - jB_m$

$Y_o = \frac{I_o}{V_1}$

$G_i = \frac{P_o}{V_1^2}$

$B_m = \sqrt{Y_o^2 - G_i^2}$

Y_o → admitance.

G_i → conductance

B_m → Inductive suceptance

12. The emf per turn for a single-phase 2200/220 V, 50 transformer is 11 V. Calculate the number of primary and secondary turns. (Apr/may 2011)

---

13. Distinguish between off-load and on-load tap-changing. (may/Jun 2011)

OFF-load (or) of-circuit tap changing means changing the tap connection after disconnecting the load for a small period of time (sec) where as no-load tap changing means changing the tap connections without disconnecting the load.

No-load (or) changing the turn ratio of a transformer is the use of of-circuit tap changer, it is required to deenergize the transformers before changing the tap.

on-load on-load tap changers are used to change the turn ratio of transformers to regulate system voltage while the transformer is delivering load.

---

14. what happens if DC supply is applied to the transformer? (may/Jun 2012)

No alternating flux is produced so no emf is produced in the secondary winding and hence no power is delivered to the load.

---

15. why all day efficiency is lower than commercial efficiency? (may/Jun 2012)

The all day efficiency depends on load cycle of the transformer which will vary the cu loss so the efficiency of all day efficiency is lower than commercial efficiency.

16. What is meant by all day efficiency in transformer? Ⓔ (Nov/Dec 2012) (Nov/Dec 2006)

The all-day efficiency of a transformer is the ratio of the total energy output (kwh) in a 24-h day to the total energy input in the same time.

$$\text{All day } \eta = \frac{\text{o/p Energy in 24 hours}}{\text{i/p Energy in 24 hours}} \times 100$$

---

17. State the advantages of auto transformer. (Apr/may 2008)

(i) $G_{TW} - G_{auto}$ = Saving of conductor material in using auto transformer.

(ii) Higher operating efficiency.

(iii) Lower reactance,

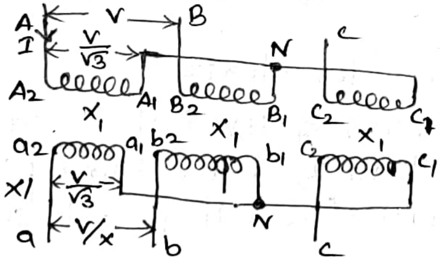
(iv) lower losses

(v) Smaller exciting current and better voltage regulation, compare to its two winding counter parts.

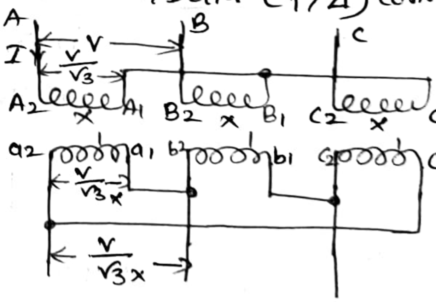
---

18. List out any four three phase transformer connections. (Nov/Dec 2012) (Nov/Dec 2006)

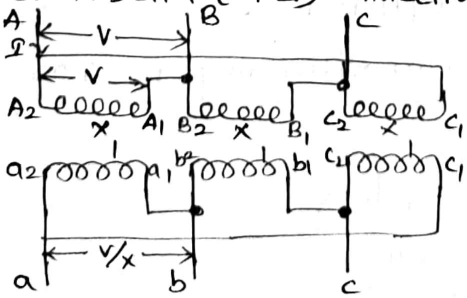
(i) star/star (Y/Y) connection



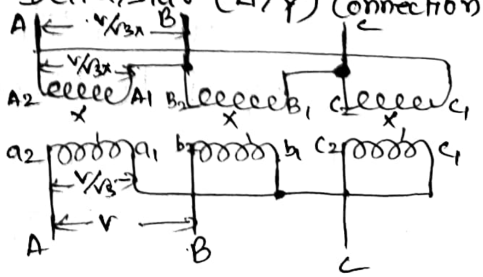
(iii) star/Delta (Y/Δ) Connection



(ii) Delta/Delta (Δ/Δ) Connection.



(iv) Delta/star (Δ/Y) Connection



19. Define regulation of a transformer. (NOV/Dec 2013)

The voltage regulation is defiend as the chang in magnitude of the secondary voltage as the load current changes from the no-load to the loaded condition.

$$\text{voltage regulation} = \frac{V_{20} - V_{2,fl}}{V_{2,fl}} \times 100$$

$V_{2,fl}$ = rated secondary voltage while supplying full load at specified powerfactor.

V_{20} = Secondary voltage when load is thrown off.

---

20. State the advantages and applications of auto transformer. (NOV/Dec 2013)

Advantages:

(i) Saving of conductor material in using auto transformer.

(ii) Lower reactance

(iii) Lower losses (The reduction in conductor and core materials, the ohmic losses in conductor and the core losses are lowered) ∴ The Auto T/f has higher efficiency than the two winding T/f of the same o/p.)

(iv) Smaller exciting current and better voltage regulation compared to its two-winding counter part.

Application:

(i) Induction motor starters.

(ii) Interconnection of HV systems at voltage levels ratio less than 2, and obtaining variable voltage power supplies (low voltage and current levels)

---

21. Give the expression for the load current when the transformer operates at its maximum efficiency. (NOV/Dec 2006)

The condition maximum efficiency is variable loss is equal to Fixed loss.

$$I_L^2 (R_{e2}) = \text{Iron loss}$$

$$\therefore I_L^2 = \frac{\text{Iron loss}}{R_{e2}}$$

$$\therefore I_L = \sqrt{\frac{\text{Iron loss}}{R_{e2}}}$$

$I_L^2 R_{e2} = W_i$

(or) Copper loss = Iron loss is the condition for maximum efficiency of transformer.

22. what are the advantages of single winding transformer over two winding transformer? (Nov/Dec 2008)

Same Answer for Q. No (17)

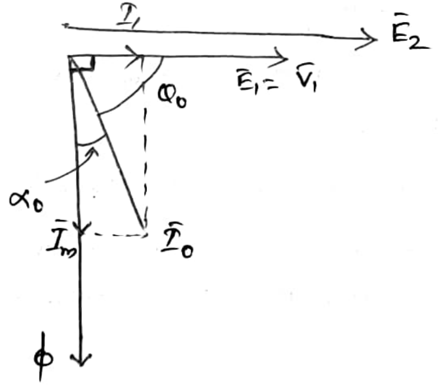
---

23. why is the efficiency of transformer more than that of rotating machines? (Nov/Dec 2012)

A transformer there is no any mechanical losses of rotation i.e friction loss, windage loss, Brush friction loss, Bearing friction loss is absent in the transformer. So that the efficiency of transformer is in the range of 96 - 99 %, more that that of rotating machines.

---

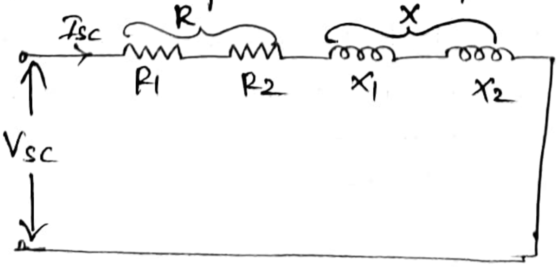
24. Draw the No-Load phasor diagram of Transformer.



---

25. which equivalent circuit parameters can be determined from the short circuit test on a transformer?

The short circuit test serves the purpose of determining the series parameters of a transformer.



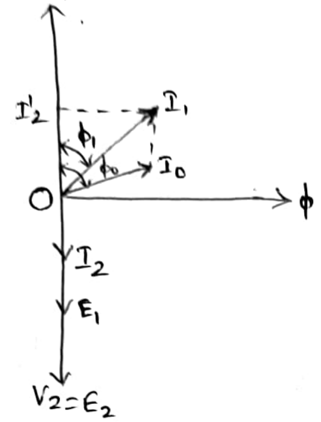
The equivalent circuit parameters are computed

$$Z = \frac{V_{sc}}{I_{sc}} = \sqrt{R^2 + X^2}$$

Equivalent resistance, $R = \frac{P_{sc}}{(I_{sc})^2}$

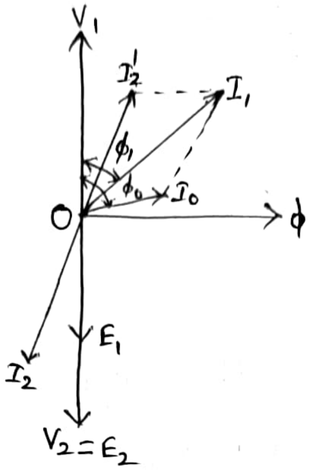
Equivalent reactance, $X = \sqrt{Z^2 - R^2}$

26. Draw the phasor diagram for a transformer on non-inductive, Inductive and capacitance loads.

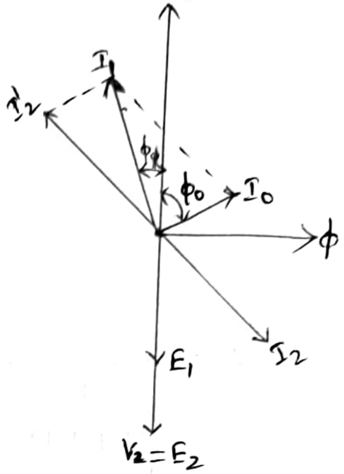


Non-Inductive load

or Resistive Load



Inductive load

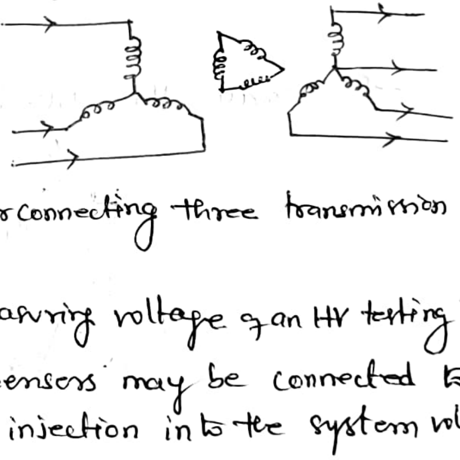


capactive load

---

27. Wha is the basic purpose of tertiary winding?

- Transformer may be built with a third winding, called tertiary, in addition to the primary and secondary.
- To supply the substation auxilaries at a voltage differ from those of the primary and secondary windings.



- Three winding may be used for interconnecting three transmission lines at different voltage level.
- Tertiary can serve the purpose of measuring voltage of an HV testing T/f.
- static capacitor or synchronous condensers may be connected to the tertiary winding for reactive power injection into the system voltage control.

---

28. Define sumpner's test.

- The sumpners test is another method of determining efficiency, regulation and heating under (actual) load conditions.
- while conducting this test, the primaries of the two identical transformer are connected in parallel across the (V₁), while the two secondaries are connected in phase opposition, The voltage across ~~ness~~ T_2 T_4 must be zero.

UNIT-II

PART-B (16 Mark Questions)

1. **Describe the construction and principle of operation of single phase transformer.** (16)

CONSTRUCTION:

TANK → The assembled Transformer with magnetic frame and windings is housed in proper tank, that contains Transformer oil and various parts of Transformer.

CONSERVATIVE → * power Transformer are provide with a conservative through which the transformer breathes into the atmosphere.
* The conservative is a smaller-sized tank placed on top of the main tank.

MAGNETIC CORE:→ The magnetic core of a transformer is made up of staks of thin lamination (0·35mm thickness) of cold-rolled grain-oriented silicon steel lightly insulated with varnish.

(i) Core TYPE: In core-type the windings are wound around the two legs of a rectangular magnetic core.

(ii) Shell TYPE: In shell type the windings are wound on the central leg of a three-legged core.

PRIMARY & SECONDARY COILS: → The primary and secondary coils usually of copper are wound on the core and are electricaly insulated from each other and from the core.

→ L·V winding: The L·V winding is wound on the insid of each limb.
→ H·V winding: The H·V winding is wound on the out side of each limb.
→ The two windings are arranged as concentric coils.

OIL DUCTS → oil ducts always provided between two solid parts, so as to provide better cooling of these parts.

BREATHER → A breather mounted on the Transformer tank contains calcium chloride or silicagel, which has the tendency to extract moisture from air.

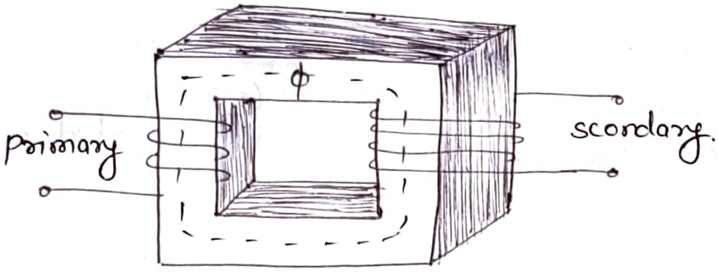
BUSHING → The terminal connections of the winding are taken to the insulator bushing mounted on the Transformer tank top, and used to isolated the current carrying part.

principle of operation:

→ A Transformer is a static device is transformed the energy from one circuit to another circuit, without change in frequency.

→ Transformer is a mutual induction between two circuits linked by common magnetic flux.

→ It consist of two inductive coils which are electrically seperated but magnetically linked through a path of low reluctance



→ The two coils posses high mutual inductance, it produces mutually - induced e.m.f (according to faradays law of electro magnetic induction, $e = M dI/dt$)

→ Electric energy is transferred (entirely magnetically) from the first coil to the second coil i.e Py to Sy.

→ The first coil, in which electric energy is fed from the a.c supply mains, is called primary winding and the other from which energy is drawn out, is called secondary winding.

2. (i) Derive the emf equation of a transformer. **(8)**

The Py winding has flux linkages.

$$\lambda_1 = N_1 \phi$$

Induced emf

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi}{dt} \rightarrow ①$$

According to kirchhoff's law.

$$v_1 = e_1 \rightarrow ②$$

let $\phi = \phi_{max} \sin\omega t \rightarrow ③$

Take derivative.

$$\frac{d\phi}{dt} = \phi_{max} \omega \cos \omega t$$

$\omega = 2\pi f$ rad/sec,

ϕ_{max} = maximum value of core flux

f = frequency of voltage source.

The emf induced in the primary winding.

$$e_1 = N_1 \frac{d\phi}{dt}$$

$$e_1 = N_1 \phi_{max} \omega \cos\omega t \rightarrow ④$$

$$e_1 = E_{max} \cos\omega t$$

from eqn ③ and ④ it is found that the induced emf leads the flux by 90^0

The rms value of the induced emf is

$$E_1 = \frac{E_{max}}{\sqrt{2}}$$

$$E_1 = \frac{\omega N_1 \phi_{max}}{\sqrt{2}}$$

$$E_1 = \frac{2\pi f N_1 \phi_{max}}{\sqrt{2}}$$

$$E_1 = \frac{\cancel{\sqrt{2}} \times \sqrt{2}\, \pi f N_1 \phi_{max}}{\cancel{\sqrt{2}}}$$

$$E_1 = \sqrt{2}\, \pi f N_1 \phi_{max}$$

$$\boxed{E_1 = 4.44 f N_1 \phi_{max}} \text{ volts} \longrightarrow ⑤$$

since $E_1 = V_1$ as per eqn ②

$$E_1 = V_1$$

$$\phi_{max} = \frac{E_1 \, (=V_1)}{4.44 f N_1} \longrightarrow ⑥$$

IIIly The emf induced in the secondary winding N_2 is given by.

$$e_2 = N_2 \frac{d\phi}{dt}$$

$$V_2 = e_2$$

$$\boxed{E_2 = 4.44 f N_2 \phi_{max}} \text{ volts}.$$

13:

3. A transformer on no-load has a core-loss of 50W, draws a current of 2A (rms) and has an induced emf of 230V (rms) .Determine the no-load power factor, core-loss current and magnetization current. Also calculate the no-load circuit parameters of the transformer. Neglect winding resistance and leakage flux. **(16)**

No load power factor, $\cos\phi_o = \dfrac{P_i}{I_o \times E_1}$

$$\cos\phi_o = \frac{50}{2\times 230} = 0.108 \text{ (lagging)}$$

$$\phi_o = \cos^{-1}(0.108) = 83.76^\circ$$

Magnetizing current, $I_m = I_o \sin\phi_o$

$$= 2\times \sin(\cos^{-1} 0.108)$$

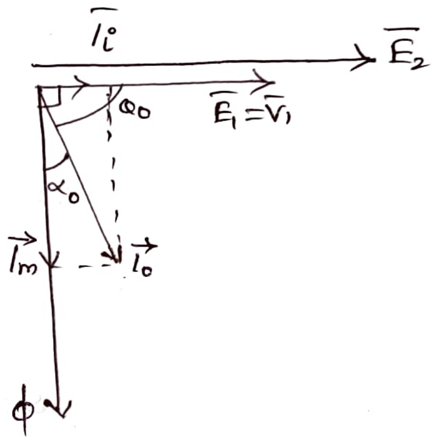
$$= 1.988 \text{ (Ampears)}$$

$\phi_o \approx 90^\circ$, there is hardly any difference between the magnitude q the exciting current and its magnetizing component.

core-loss current, $I_i = I_o \cos\phi_o$

$$I_i = 2\times 0.108$$

$$I_i = 0.216\text{A}$$



No-load circuit parameter q the T/f

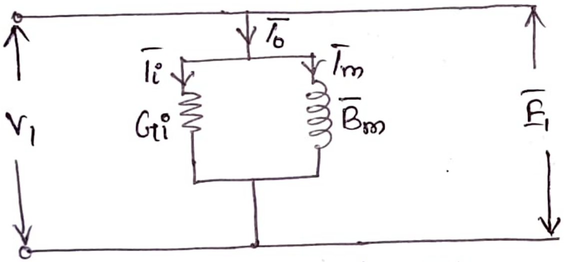


fig- circuit model q T/f on no-load (exciting current)

Core loss, $P_i = G_i V_1^2$ (or) $G_i = \dfrac{P_i}{V_1^2}$

$$G_i = \frac{P_i}{V_1^2} = \frac{50}{(230)^2} = 9.45\times 10^{-4}$$

$I_m = B_m V_1$ (or) $B_m = \dfrac{I_m}{V_1}$

$$B_m = \frac{1.988}{230} = 8.64\times 10^{-3}\ \mho$$

4. **The following data were obtained on a 20 KVA,50 Hz, 2000/200 V distribution transformer:**

	Voltage (V)	Current (A)	Power (W)
OC test with HV open-circuited	200	4	120
SC test with LV short-circuited	60	10	300

Draw the approximate equivalent circuit of the transformer referred to the HV and LV sides respectively. **(16)**

OC test (LV side)

$$Y_0 = \frac{I_0}{V_1} = \frac{4}{200} = 0.02\,\mho$$

$$G_i = \frac{P_0}{V_1^2} = \frac{120}{(200)^2} = 3\times10^{-3}\,\mho$$

$$B_m = \sqrt{Y_0^2 - G_i^2}$$

$$B_m = \sqrt{(0.02)^2 - (3\times10^{-3})^2}$$

$$B_m = 0.01977\,\mho$$

SC test (HV side)

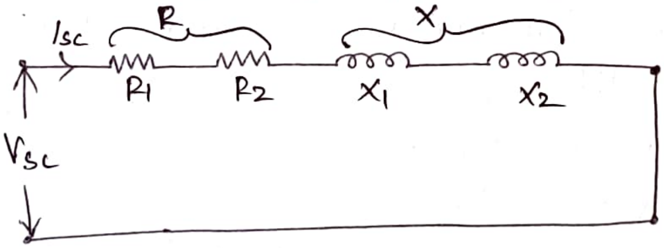
$$Z = \frac{V_{sc}}{I_{sc}} = \frac{60}{10} = 6\,\Omega$$

$$R = \frac{P_{sc}}{(I_{sc})^2} = \frac{300}{(10)^2} = 3\,\Omega$$

$$X = \sqrt{Z^2 - R^2} = \sqrt{6^2 - 3^2} = 5.2\,\Omega$$

Transformation ratio, $\frac{N_H}{N_L} = \frac{2000}{200} = 10$

i.e. $P_{sc} = P_c$ (copper - loss)



$$Z = \frac{V_{sc}}{I_{sc}} = \sqrt{R^2 + X^2}$$

Equivalent resistance $R = \frac{P_{sc}}{(I_{sc})^2}$

Equivalent reactance $X = \sqrt{Z^2 - R^2}$

Note! The OC and SC test together give the parameters of the approximate equivalent circuit.

TAP CHANGING TRANSFORMER

1. A tap changer is a device fitted to power transformers for regulate the output voltage to required levels.
2. This is normally achieved by changing the ratios of the transformers on the system by altering the number of turns in one winding of the appropriate transformer/s.
3. Adjustment of consumer terminal voltage with in prescribed limits.
4. Adjust is normally carried out by off-circuit tap changing the common range being 5% in 2.5%steps.
5. Daily and short-time control or adjustment is carried out by means of on-load tap changing gear.

Tap changing transformer also provide for the following purposes:

1. For varying the secondary voltage.
2. For maintaining the secondary voltage constant with varying primary voltage.
3. For providing an auxiliary secondary voltage for the special purpose, such as lighting.
4. For providing a low voltage for starting rotating machines.
5. For providing a neutral point, e.g. for earthing.

Location:

1. The taps may be placed on the primary or secondary side which partly depends on construction.
2. If tapings are near the line ends, fewer bushing insulators are required.

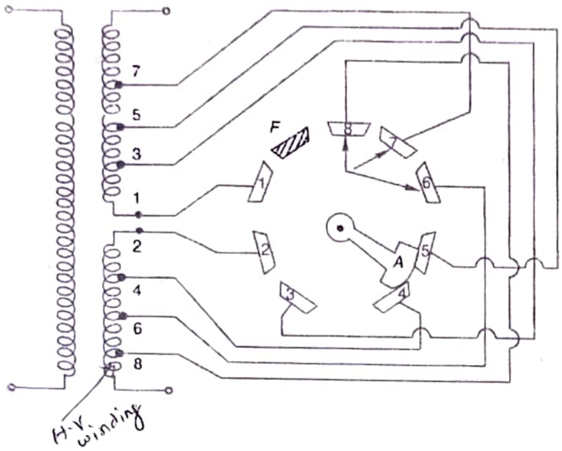
Types of tap changing transformer:

(i) NO-load (off-load or off-circuit) tap changing.

(ii) ON-load tap changing.

NO-load tap changing:

1. The cheapest method of changing the turn ratio of a transformer is the use of off-circuit tap changer.
2. It is required to de energize the transformer before changing the tap.



NO-load tap changer

Construction:

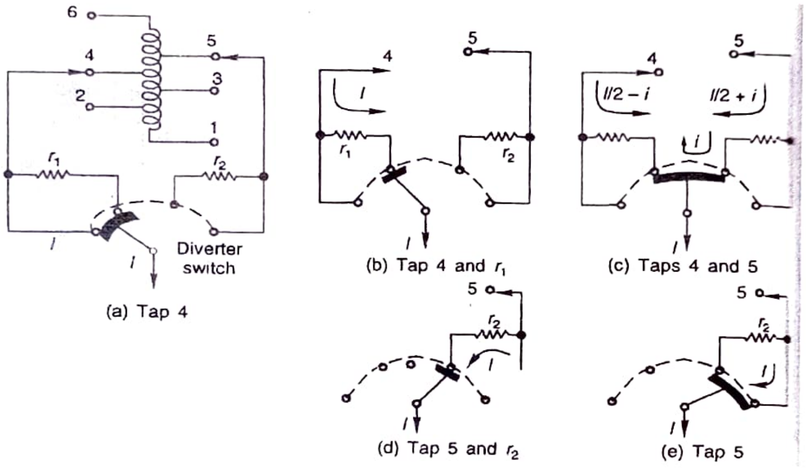
1. It has eight studs marked 1 to 8.the winding is taped at eight points.
2. The face plate carrying the suitable studs can be mounted at a convenient place on the transformer such as upper yoke.
3. The movable contact arm 'A' may be rotated by hand wheel mounted externally on the tank.
4. The winding is tapped at 2% intervals, then as the rotatable arm 'A' is moved over to stud 1,2; 2,3; 3,4;4,5;5,6;6,7;7,8.
5. The stop 'F' which fixes the final position of the arm 'A' prevents further anti clock wise rotation ,so that stud 1 and 8 cannot be bridged by the arm.
6. Adjustment of tap setting is carried out with transformer de energized.

Advantages:

1. To prevent unauthorized operation of an off-circuit tap changer, a mechanical lock is provided.
2. Further, to prevent inadvertent operation, an electromagnetic latching device.

ON-load tap changing:

1. On-load tap changers are used to change the turn's ratio of transformer to regulate system voltage while the transformer is delivering load.
2. The operating efficiency of electrical system gets considerably improved.
3. On-load tap changing circuits are provided with impedance, which is introduced to limit short circuit current during the tap changing operation.
4. The impedance can either be a resistor or centre taped reactor.



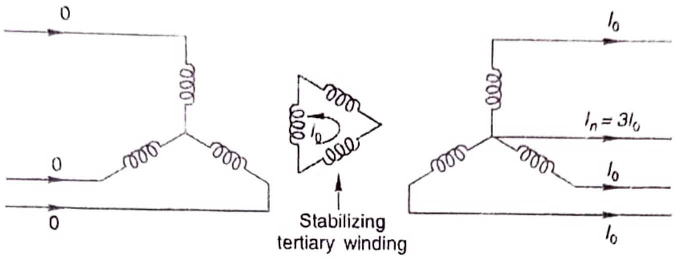
(a) Tap 4 (b) Tap 4 and r_1 (c) Taps 4 and 5 (d) Tap 5 and r_2 (e) Tap 5

The sequence of operation during the transient from one tap to next (adjoining)

1. On-load –tap changing system is has two components.
   (a) Diverter switch.
   (All the transient operation of switching is performed at the diverter switch)
   (b) Selector switch.
   (The selector switches are arranged on the tapings 1, 2,3,4,5 as shown in the figure.
2. Usually the diverter switch is kept in a separate chamber, filled up with oil.
3. The diverter switch is connected between A and B .the selector switch 4 is under action at this moment then the diverter switch arm is shifted from 'AB' to 'B' only.
4. Therefore, the winding is connected through resistance r_1, thus the current is reduced.
5. Now we want to change the tap from 4 to 5 without disconnecting the supply.
6. The diverter switch is moved from 'B' to BC .so the current is diverted in to $r_1$and r_2 then the diverter switch arm is shifted from 'BC' to 'C' switch 5 is under action at this moment.
7. In the above process the load is not disconnected (On-load tap changers).
8. The aim is to maintain a given voltage level within a specified tolerance.

TERTIARY WINDING (THREE WINDING TRANSFORMERS)

1. Transformers may be built with a third winding, called tertiary, in addition to the primary secondary.



The uses of tertiary winding are enumerated below:

(i) To supply the substation auxiliaries at a voltage differ from those of the primary and secondary windings.

(ii) Static capacitor or synchronous condensers may be connected to the tertiary winding for reactive power injection into the system voltage control.

(iii) Three winding may be used for interconnecting three transmission lines at different voltage level.

(iv)Tertiary can serve the purpose of measuring voltage of an HV testing transformer.

Sumpner's Test or (Back to Back Test)

- The Sumpner's test is another method of determining efficiency, regulation and heating under load conditions.

- The O.C. and S.C. tests give us the equivalent circuit parameters but cannot gives heating information under various load conditions.

- The Sumpner's test gives heating information also.

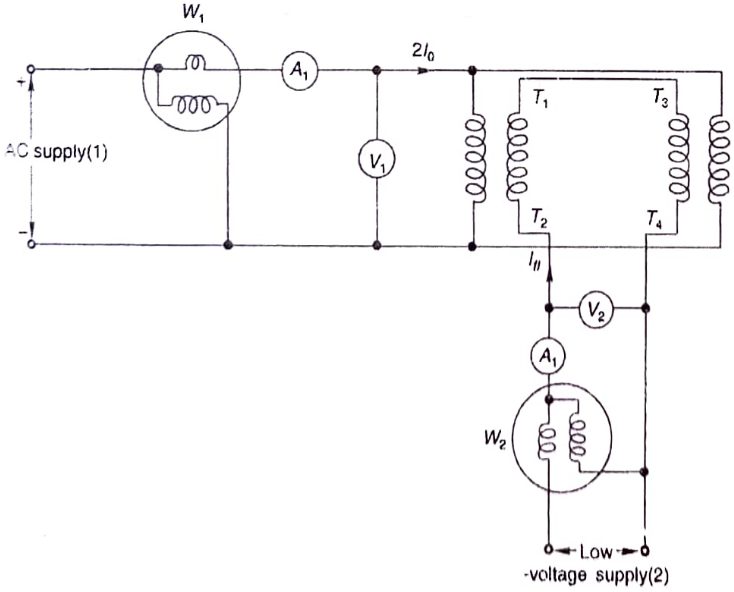
- In O.C. test, there is no load on the transformer while in S.C. circuit test can be loaded fictitiously (imaginary or not real) to the full-rated condition without actual loading the transformer to its full load.

- In all in O.C. and S.C. tests, the loading conditions are absent. Hence the results are inaccurate.

- In Sumpner's test, actual loading conditions are simulated hence the results obtained are much more accurate.

- Thus Sumpner;s test is much improved method of predetermining regulation and efficiency than O.C. and S.C. tests.

Circuit diagram:



(Fig-a)Sumpner 's test on two identical single-phase transformers

- While conducting this test, the primaries of the two identical transformers are connected in parallel across the supply (V_1), while the two secondaries are connected in phase opposition as shown in the figure (Fig-a).

- For the secondariec to be in phase opposition, the voltage across T_2T_4 must be zero otherwise it will be double the rated secondary voltage in which case the polarity of one of the secondaries must be reversed.

- Current at low voltage (V_2) is injected in to the secondary circuit at T_2T_4.

- As per the super position theorem, if V_2 source is assumed shorted, the two transformers appear in open-circuit to source V_1as their secondaries are in phase opposition and therefore no current can can flow in them.

- The current drawn from the source V_1 is thus $2I_0$ (twice the no-load current of each transformer) and power is $2P_0$ (=$2P_i$,twise the core –loss of each transformer).

- When V1 is regarded as shorted, the transformers are series –connected across V_2 and are short –connected on the side of primaries.

- Therefore ,the impedance seen at V_2 is 2Z and when V_2 is adjusted to circulate full-load current(I_{fl}) ,the power fed in is $2P_C$(twice the full load copper-loss of each transformer).

- Thus in the sumpner's test while the transformers are not supplyine any load ,full iron –loss occurs in their cores and full copper –loss occurs in their windings ; net power input to the transformers being being ($2P_0$+$2P_C$).

- The heat run test could, therefore, be conducted on the two transformers, while only losses are supplied.

- In (Fig-a) the auxiliary voltage source is included in the circuit of secondaries ; the test could also be conducted by including the auxiliary source in the circuit of primaries.

ADVANTAGES:

- Thus in the sumpner's test without supplying the load, full iron loss occurs in the core while full copper loss occurs in the windings are measured simultaneously.

- The power required to carry out the Sumpner 's test is small.

DISADVANTAGES:

- The sumpner's test required two same rating transformers compare to O.C. and S.C. tests.

Prepared by
V.SHANMUGAM, M.E,
Assistant professor (EEE)

Parallel Operation of Transformers

Introduction:

- For supplying a load in excess of the rating of an existing transformer, two or more transformers may be connected in parallel with the existing transformer.

- The transformers are connected in parallel when load on one of the transformers is more than its capacity.

- A half the load can be supplied with one transformer out of service.

Condition for Parallel Operation of Transformer:

- For parallel connection of transformers, primary windings of the Transformers are connected to source bus-bars and secondary windings are connected to the load bus-bars.

- Various conditions that must be fulfilled for the successful parallel operation of transformers:

1. Same voltage Ratio & Turns Ratio (both primary and secondary Voltage Rating is same).

2. Same Percentage Impedance and X/R ratio.

3. Identical Position of Tap changer.

4. Same KVA ratings.

5. Same Phase angle shift (vector group are same).

6. Same Frequency rating.

7. Same Polarity.

8. Same Phase sequence.

- Some of these conditions are convenient and some are mandatory.

- The convenient are: Same voltage Ratio & Turns Ratio, Same Percentage Impedance, Same KVA Rating, Same Position of Tap changer.

- The mandatory conditions are: Same Phase Angle Shift, Same Polarity, Same Phase Sequence and Same Frequency.

- When the convenient conditions are not met paralleled operation is possible but not optimal.

Testing:

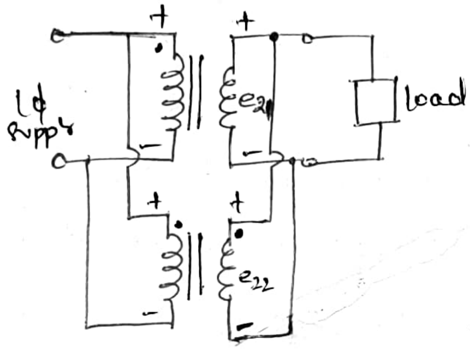
(i) polarity test.

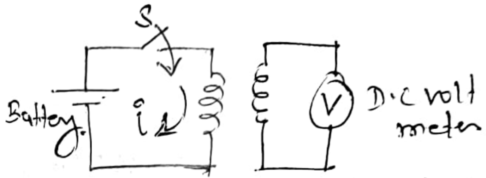
→ windings on T/f are marked to indicate terminals of polarity.

→ They indicate how the windings are wound on the core.

→ polarities of windings must be known if Transformers are connected in parallel to share a common load.

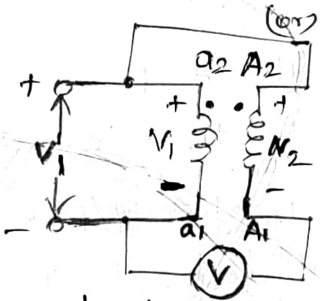
Eg!





A simple polarity test.

(or)

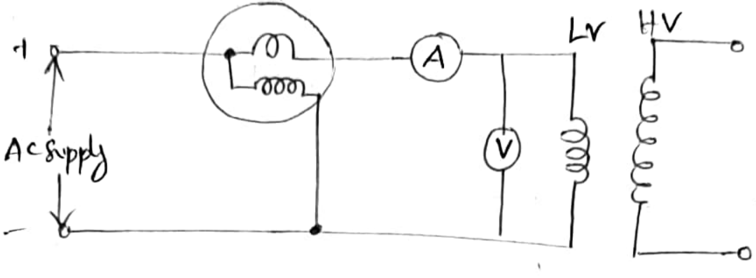


polarity test on two wdg T/f

→ If the polarities of the windings are as marked on the diagram, the voltmeter should read $V = V_1 \sim V_2$. If it read $V = V_1 + V_2$, the polarity marking of one of the wdg must be interchanged.

(ii) open circuit test (oc) or NO-load Test.

→ The purpose of this test is to determine no-load losses, and shunt branch parameters of the equivalent circuit of the T/f.

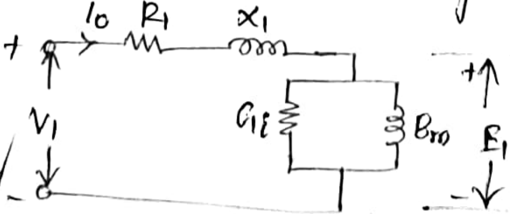


- * one of the wdg is connected to supply at rated voltage, while the other wdg is kept open-circuited.
- * The test is usualy performed from the LV side, while the HV side is kept open circuited.
- * If the T/f is to be used at voltage other than rated, the test should be carried out at that voltage
- * Meters are arranged to read.

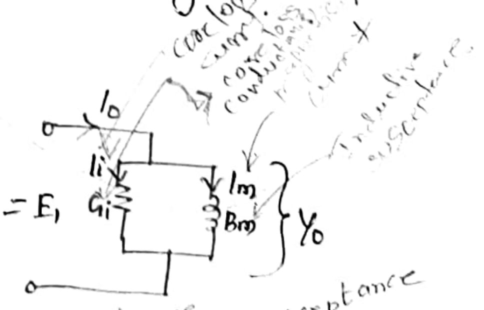
  voltage $= V_1$; current $= I_0$ and power I/p $= P_0$
- * Indeed the (no-load current I_0 is so small (it is usually 2-6% of the rated current)) and R_1 and x_1 are also small, that V_1 can be regarded as $= E_1$ by neglecting the series impedance. This means that for all practical purpose the power i/p on no-load equal the core (iron) loss i.e $P_0 = P_i$ (iron-loss)
- * The shunt branch parameters can easily be determined from the three readings.

$W = V_1 I_0 \cos\phi$

∴ The OC test yield the values of core-loss and parameters of shunt branch of the equivalent circuit.



$E_1 \approx V_1 = E_1$



exciting admitance

$$Y_0 = G_i - jB_m$$

now

$$Y_0 = \frac{I_0}{V_1}$$

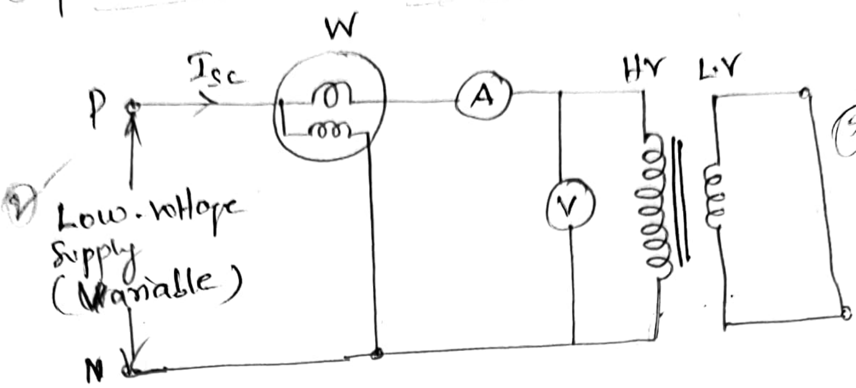
exciting conductance   exciting susceptance

$$V_1^2 G_i = P_0 \quad \text{or} \quad G_i = \frac{P_0}{V_1^2}, \quad B_m = \sqrt{Y_0^2 - G_i^2}$$

SHORT CIRCUIT TEST: (or) Impedance Test.

→ The purpose of this test to determining the series ① parameters of a Transformer and full load copper loss.

(This loss is used in calculating the η of the T/f.)



② Low-voltage Supply (Variable)

⑤ ~~determining~~
(1) equivalent impedance Z_{01}, Z_{02}
(2) equivalent ~~leakage~~ reactance X_{01}, X_{02}
(3) equivalent resistance R_{01}, R_{02} of the T/f
Knowing Z_{01} or Z_{02}

→ ③ The test is usually, conducted from The H.V side of the T/f while the L.V is short circuited, by a thick conductor (or through an ammeter which may serve the additional purpose of indicating rated load current)

→ voltage needed for the short circuited test is typically 5% of the rated value. and voltage and current to be handled

Eg: 200 kVA, 440/6600-V T/f test on the H.V side would require

$$\frac{6600 \times 5}{100} = 330V. \text{ and}$$

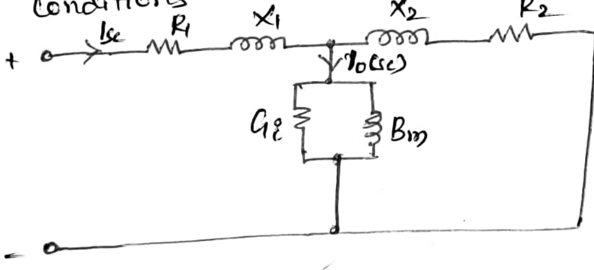
$$\frac{200 \times 1000}{6600} = 30 A \text{ Supply.}$$

while if conducted from the L.V side it would need.

$$\frac{440 \times 5}{100} = 22V \text{ and}$$

$$\frac{200 \times 1000}{440} = 455 A$$

→ ⑥ The equivalent circuit as shown below the HV under short circuited conditions



voltage = V_{sc} : Current = I_{sc}

power i/p = P_{sc}

→ Since T/F resistance and leakage reactance are very small, the voltage V_{sc} need to circulate the full load current under short-circuit is as low a 5-8% of the rated voltage.

→ under these condition $I_{0(sc)}$ Exciting current is only about 0.1 to 0.5% of the full-load current

→ ④ The S.C test, the supply voltage is gradually raised from zero till the Transformer draws full-load current.

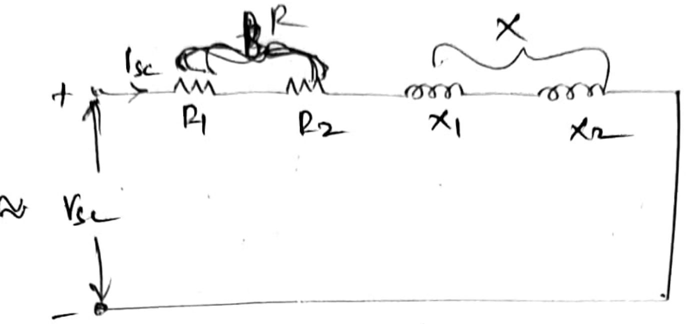
V_{sc}, I_{sc}, P_{sc}

→ The T/f excited at very low voltage, the iron-loss is negligible (That is why the shunt branch resistance is left out), The power i/p corresponds only to the copper loss.

i.e

$$\overset{\text{power i/p}}{P_{sc}} = P_c \text{ (Copper - loss)}$$

ⓑ



$$Z = \frac{\overset{\text{voltage}}{V_{sc}}}{\underset{\text{current}}{I_{sc}}} = \sqrt{R^2+X^2}$$

Equivalent resistance $R = \dfrac{P_{sc}}{(I_{sc})^2}$

Equivalent reactance $X = \sqrt{(Z^2 - R^2)}$

It was observed that oc and sc test together give the parameters of the approximate equivalent circuit (which is already pointed out) i[illegible]

problem: (Related to O/c & S/c)

① The following data were obtained on a 20 KVA, 50 Hz, 2000/200 V distribution T/f:

Supply given to LV side

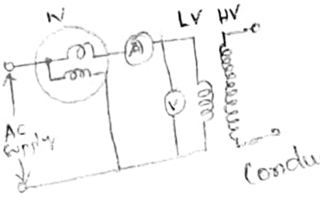
	voltage (V)	current (A)	power W
OC test with HV open-circuited	200	4	120
SC test with LV short-circuited	60	10	300

supply given to HV side.

Draw the approximate equivalent circuit of the T/f referred to the HV and LV sides respectively.

Solution:

OC test (LV side)



Exciting admittance (or) shunt branch admittance

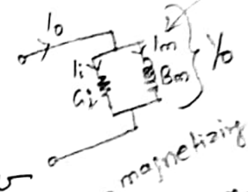
no load current

$$Y_0 = \frac{I_0}{V_1} = \frac{4}{200} = 0.02\ \mho$$

No load power

Conductance

$$G_i = \frac{P_0}{V_1^2} = \frac{120}{(200)^2} = 3\times10^{-3}\ \mho$$



magnetizing current

magnetizing current

$I_m = I_0 \sin\phi_0$

$I_i = I_0 \cos\phi_0$

core loss current

susceptance (or) exciting susceptance

$$B_m = \sqrt{Y_0^2 - G_i^2}$$

$$B_m = \sqrt{(0.02)^2 - (3\times10^{-3})^2} = \sqrt{3.91\times10^{-4}} = 0.01977\ \mho$$

SC Test (HV side)

Impedance

$$Z = \frac{V_{sc}}{I_{sc}} = \frac{60}{10} = 6\,\Omega$$

equivalent resistance

Short circuit power

$$R = \frac{P_{sc}}{(I_{sc})^2} = \frac{300}{(10)^2} = 3\,\Omega$$

short circuit current

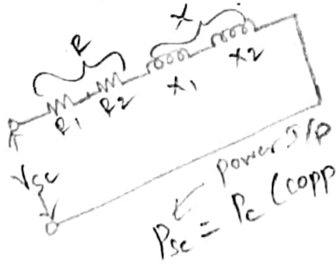
equivalent reactance.

$$X = \sqrt{Z^2 - R^2} = \sqrt{6^2 - 3^2} = 5.2\,\Omega$$

Transformation ratio, $\frac{N_H}{N_L} = \frac{2000}{200} = 10$

Note

$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$



power I/p

$P_{sc} = P_c$ (copper loss)

2·30

equivalent circuit referred to the HV side:

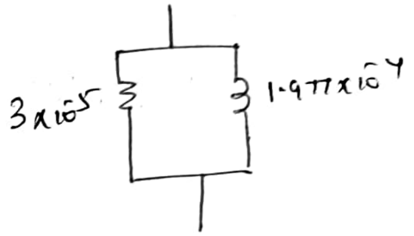
$$G_i(HV) = G_i(LV) \times \frac{LV(\text{voltage})}{HV(\text{voltage})}$$

$$= \cancel{0.8\times10} \; 3\times10^{-3} \times \left(\frac{200}{2000}\right)^2 = 3\times10^{-5} \; ℧$$

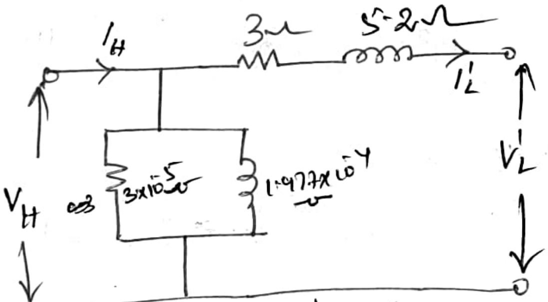
$$B_m \cancel{B} (HV) = B_m(LV) \times \frac{Lv(\text{voltage})}{Hv(\text{voltage})}$$

$$= 0.01977 \times \left(\frac{200}{2000}\right)^2$$

$$= 1.977\times10^{-4} \; ℧$$



equivalent circuit



Refferred to HV

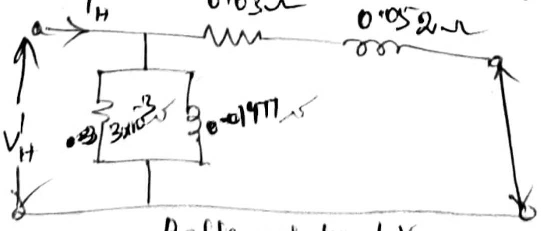
equivalent circuit referred to the ~~HV~~ LV side:

$$R(LV) = R_{HV} \times \left(\frac{Lv(\text{volts})}{Hv(\text{volts})}\right)^2$$

$$R(LV) = 3 \times \left(\frac{200}{2000}\right)^2 = \cancel{0.3\times10^{-4}} \; 0.03 \; Ω$$

$$X(LV) = X_{HV} \times \left(\frac{Lv(\text{volts})}{Hv(\text{volts})}\right)^2$$

$$= 5.2 \times \left(\frac{200}{2000}\right)^2 = 0.052 \; Ω$$



Refferred to LV

Example (21)

The parameter of the equivalent circuit of a 150 kVA, 2400/240 V T/f are:

$R_1 = 0 \cdot 2\,\Omega$ $\quad R_2 = 2 \times 10^{-3}\,\Omega$

$X_1 = 0.45\,\Omega$ $\quad X_2 = 4.5 \times 10^{-3}\,\Omega$

$R_i = 10\,k\Omega$ $\quad X_m = 1.6\,k\Omega$

note (as seen from 2400V side) of i.e H.V side

Calculate:

(a) open-circuit current, power and p.f when L.V is Excited at rated voltage.

(b) The voltage at which the HV should be excited to conduct a short-circuit (LV shorted) with full load current flowing. what is the i/p power its pf?

Solution

$$\text{Ratio transformation } a = \frac{V_1}{V_2} = \frac{\text{Hv side}}{\text{Lv side}} = \frac{2400}{240} = 10$$

(a) Referring the shunt parameters to Lv side.

iron loss equivalent resistance → $R_i\,(LV) = R_i\,(\text{HV side}) \times \left(\frac{\text{Lv side voltage}}{\text{Hv side voltage}}\right)^2$

$$= 10{,}000 \times \left(\frac{240}{2400}\right)^2 = 100\,\Omega$$

$$X_m(LV) = X_m\,(\text{Hv side}) \times \left(\frac{\text{Lv side voltage}}{\text{Hv side voltage}}\right)^2$$

$$= 1600 \times \left(\frac{240}{2400}\right)^2 = 16\,\Omega$$

$I = \frac{V}{R}$

$$\bar{I}_0\,(LV) = \frac{V_2}{R_i} - j\frac{V_2}{X_m}$$

because it is inductive current ↑ it is always lagging.

$$= \frac{240\angle 0^\circ}{100} - j\,\frac{240\angle 0^\circ}{16}$$

$$= 2.4 - j15 = 15.2\angle -80.9^\circ\,A$$

$P_0 = V_0 I_0 \cos\phi_0$

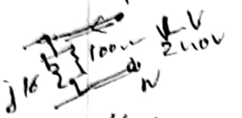
Power = 240 × 15.2 × 0.158
= 576.384 watts

$I_0 = 15.2\,A$, $pf = \cos 80.9^\circ = 0.158$ lagging

$\text{power} = V_0 I_0 \cos\phi$

$P_0 = 240 \times 15.2 \times 0.158$

$P_0 = 0.576 \times 10^3$ or 0.576 kW



$I_w = \frac{240}{100\Omega} = (2.4 + j0)$

$I_\mu = \frac{240}{16} = \frac{60}{4} = (0 - j15A)$

(b) LV shorted, HV excited, full-load current flowing: shunt parameters can be ignored under this condition. Equivalent series parameters referred to HV side:

$R = R_1 + R_2'$

$R_2' = R_2 \times \left(\frac{\text{Hv side voltage}}{\text{Lv side voltage}}\right)^2$

$R = 0.2 + 2\times10^{-3} \times \left(\frac{2400}{240}\right)^2$

$R = 0.4\,\Omega$

$X = X_1 + X_2'$

$X_1' = X_2 \times \left(\frac{\text{Hv side voltage}}{\text{Lv side voltage}}\right)^2$

X_2'

$X = \overset{X_1}{0.45} + 4.5\times10^{-3} \times \left(\frac{2400}{240}\right)^2$

$X = 0.9\,\Omega$

$\bar{Z} = R + jX$

$\bar{Z} = 0.4 + j\,0.9 = 0.958 \angle 66^\circ\ \Omega$

$I_{fl}(\text{HV}) = \frac{\text{KVA}}{\text{Hv side voltage}} = \frac{150\times1000}{2400} = 62.5\text{A}$

$V_{sc}(\text{Hv}) = I_{fl} \times Z$

Power = 60 X 62.5 X .407 watt

$V_{sc}(\text{HV}) = 62.5 \times 0.958 = 59.9\text{V}$ (or) 60 V (say)

$P_{sc} = I^2 \times R = (62.5)^2 \times 0.4 = 156$ kW

$\text{Pf}_{sc} = \cos 66^\circ = 0.407$ lagging.

"All - Day" Efficiency.

- The T/f 's used for distribution are energized to the line for all the 24 hours q the day. Thus constant losses occur in the transformer for the hde day.

- These T/f 's are normally operating on a varying load during 24 hours q a day.

- As such copper losses are different during different periods q the day.

- Hence the efficiency q such transformers should be measured on the energy basic.

All day efficiency q the T/f is given by.

$$\text{All day efficiency} = \frac{\text{output in kilowatt-hours}}{\text{Input in kilowatt-hours.}}$$

(note: for 24 hours)

- This efficiency is always less than the commercial efficiency q a Transformer.

Efficiency! (T/f)

- The efficiency q a transformer is defiend as the ratio q the useful power output to the input power. Thus.

$$\eta = \frac{\text{o/p power}}{\text{i/p power}}$$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_i + I_2^2 R_{eq(2)}} \text{ (or) } \frac{V_s I_s \cos\phi}{V_s I_s \cos\phi + W_c + (I_p^2 R_p + I_s^2 R_s)}$$

W_i (or) P_i -> iron loss

$\cos\theta_2$ = load pf

V_2 -> is a rated sy voltage.

P_c (copper loss in the two winding are) $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{eq(2)}$

wher $R_{eq(2)}$ = equivalent resistance referred to the secondary side.

note: η q the T/f is much higher than that q rotating machinery, because there are no friction and windage losses.

Voltage Regulation:

* The regulation of a transformer is defined as the change of secondary terminal voltage between no load and full load conditions expressed as a percentage of the secondary ~~no~~ full load voltage, The primary voltage be in assumed constant.

$$\% \text{ voltage regulation} = \frac{(V_{20} - V_{2,fl})}{V_{2,fl}}$$

where, $V_{2,fl}$ = rated secondary voltage while supplying full load at specified powerfactor.

V_{20} = secondary voltage when load is thrown off.

Merit's of voltage Regulation

* Is used to identify this characteristic of voltage change in a Transformer with loading.

* To reduce the magnitude of the voltage change the T/f should be designed for a low value of impedance. Z_q.

Example: ①

A 20-KVA, 50HZ, 2000/200-V distribution T/f has a leakage impedance of $0{\cdot}42 + j0{\cdot}52\,\Omega$ in the high-voltage (HV) winding and $0{\cdot}004 + j0{\cdot}005\,\Omega$ in the low-voltage (LV) winding. when seen from the LV side; the shunt branch admittance Y_o is $(0{\cdot}002 - j0{\cdot}015)\,℧$ (at rated voltage and frequency). Draw the equivalent circuit to (a) HV side and (b) LV side, indicating all impedance on the circuit.

Solution:

H·V side will be reffered ~~to~~ as ~~side~~ 1 and LV side as 2.

(no load voltage ratio) check it.

$$\text{Transformation ratio}, a = \frac{N_1}{N_2} = \frac{2000}{200} = 10$$ (ratio of rated voltages: see eqn (4 in pg 2·16)

← HV side

← L·V side

(a) Equivalent circuit referred to HV side (side 1)

(we come sy to py so we give Impedance to py so we should consider $\frac{N_1}{N_2} = a$)

$$\bar{Z}'_2 = a^2(\bar{Z}_2)$$

$$\bar{Z}'_2 = (10)^2(0{\cdot}004 + j0{\cdot}005)$$

$$= 0{\cdot}4 + j0{\cdot}5\,\Omega$$

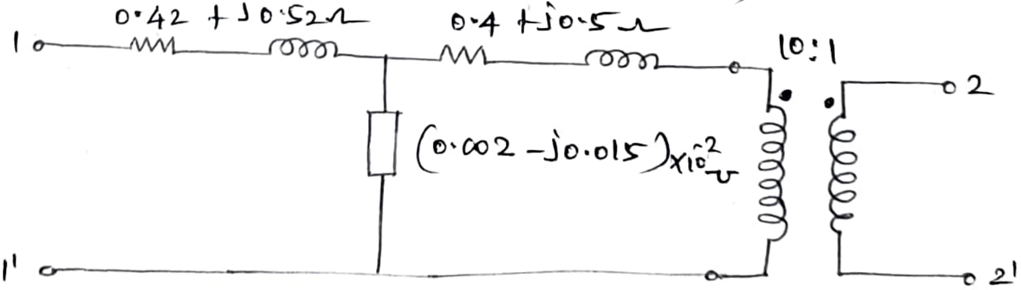
(note: Transforming admittance is divided by a^2)

$$\bar{Y}'_o = \frac{1}{a^2}(Y_o)$$ ← Shunt branch admittance.

(where Y_o = shunt branch admittance reffered to sy)

$$= \frac{1}{(10)^2}(0{\cdot}002 - j0{\cdot}015)$$

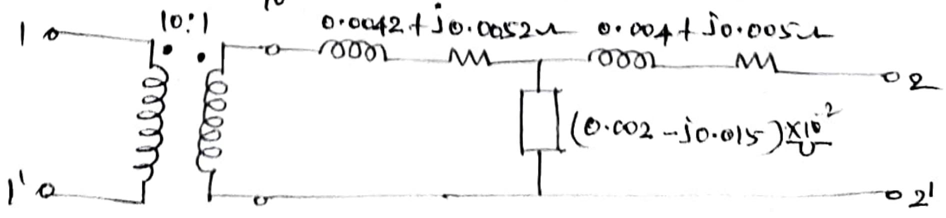
$Y_o = G - jB$



(b) equivalent circuit referred to LV side (side 2)

$$\bar{Z}'_1 = \frac{1}{a^2}\bar{Z}_1$$ ← reffer to Impedance transfer.

$$= \frac{1}{10^2}(0{\cdot}42 + j0{\cdot}52) = 0{\cdot}0042 + j0{\cdot}0052$$



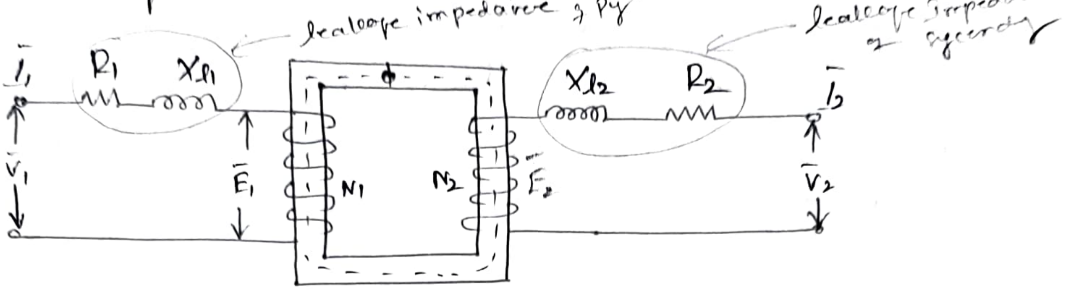
6. **Explain in detail step by step the procedure to draw the equivalent circuit of transformer.** **(16)**

P: 25
Ef: 18
P.S. Bimln

→ An equivalent circuit model based on physical reasoning

→ It will provide the same performance characteristics of the practical Transformer.

→ The equivalent circuit approach provides a better understanding



fig(a)

$\bar{I}_1$ → current flowing in the primary of the T/f

$\bar{I}_2$ → current flowing in the sy of the T/f

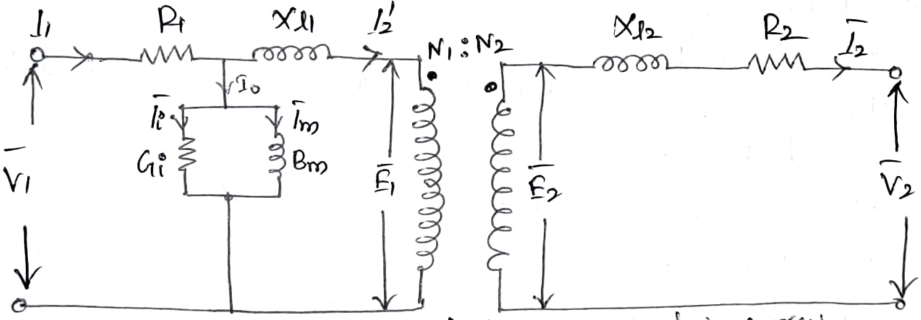
R_1, R_2 → Primary & secondary winding resistance.

X_{l1}, X_{l2} → Leakage reactance of Py & Sy.

ϕ → mutual (or common) flux.

N_1, N_2 → No of turns Py & Sy

$\bar{E}_1, \bar{E}_2$ → Induced emf (rms) of the Transformer.



fig(b)

$$\underset{\text{no-load current}}{\bar{I}_0} = \underset{\text{magnetising current}}{\bar{I}_m} + \underset{\text{core-loss current}}{\bar{I}_e}$$

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2' \qquad \text{where } \frac{\bar{I}_2'}{\bar{I}_2} = \frac{N_2}{N_1}$$

I_e → core-loss current

B_m → Inductive susceptance

G_i → conductance

(Resistance) on sy side of the T/f can be now referred to its py side

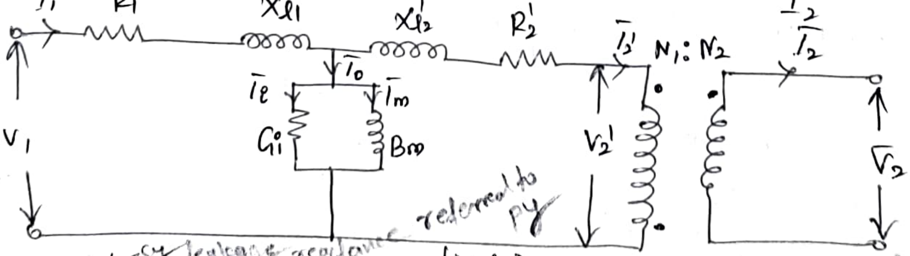


fig (c)

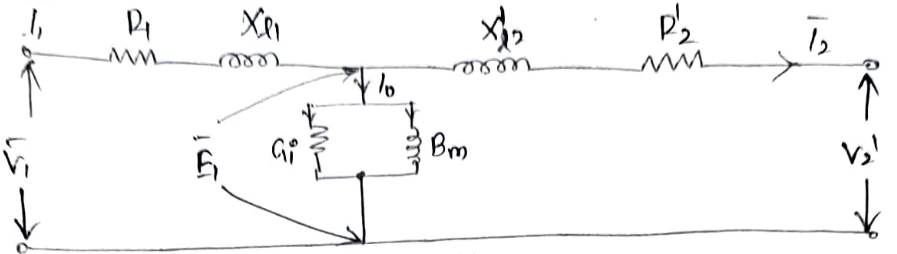
where, $X_{l2}' = \left(\frac{N_1}{N_2}\right)^2 X_{l2}$ ← sy leakage reactance referred to py

$R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2$ ← sy resistance referred to primary

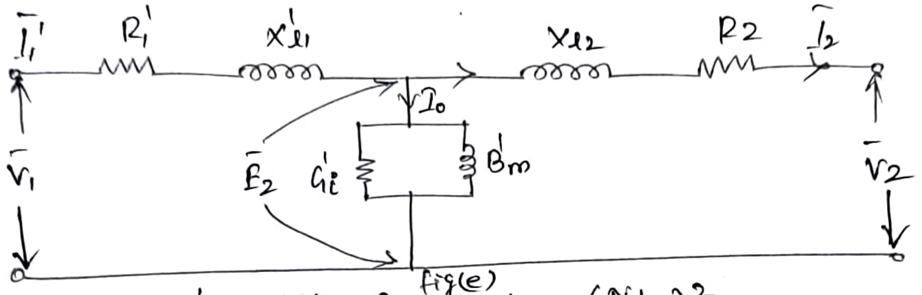
$\bar{V}_2' = \left(\frac{N_1}{N_2}\right)\bar{V}_2$, $\bar{I}_2' = \left(\frac{N_2}{N_1}\right)\bar{I}_2$

IIly $R_1' = \left(\frac{N_2}{N_1}\right)^2 R_1$ ← py resistance referred to secondary

IIly $X_{l1}' = \left(\frac{N_2}{N_1}\right)^2 X_{l1}$ ← py leakage reactance referred to sy



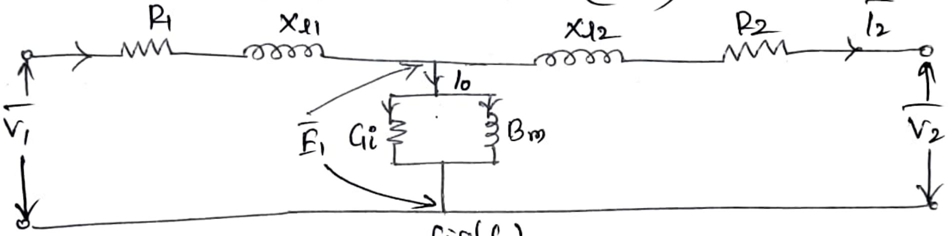
fig(d)



fig(e)

where $G_i' = \left(\frac{N_1}{N_2}\right)^2 G_i$; $B_m' = \left(\frac{N_1}{N_2}\right)^2 B_m$

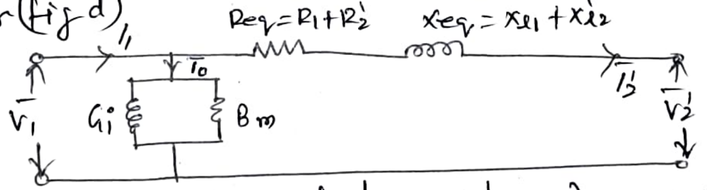
$R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2$; $x'_{l1} = \left(\frac{N_2}{N_1}\right)^2 x_{l1}$



fig(f)

Approximate Equivalent circuit

Reffer (fig d)



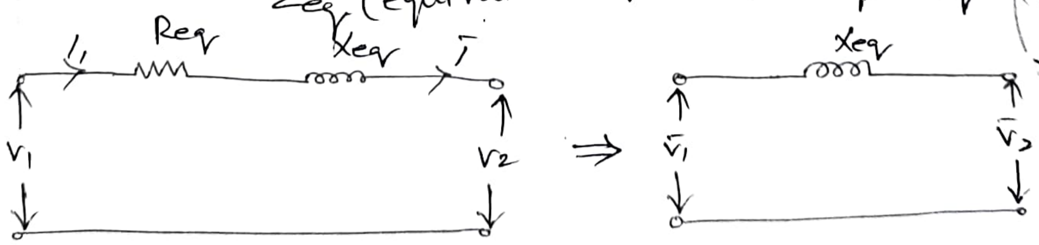
$R_{e2} = R_2 + R_1'$ equivalent resistance referred to sy

Req (equivalent resistance) = $R_1 + R_2' = R_{e1}$ ← equivalent resistance referred to py

Xeq (equivalent reactance) = $X_{l1} + X'_{l2} = X_{e1}$ ← equivalent leakage reactance referred to py

Zeq (equivalent impedance) = $R_{eq} + j X_{eq}$

$X_{e2} = X_{l2} + X'_{l1}$ equivalent leakage reactance referred to sy



TRANSFORMER LOSSES:

→ The Transformer has no moving part so that its efficiency is much higher than that of rotating machines.

(The various losses in a Transformer are enumerated below)

core loss → These are hysteresis and eddy-current losses resulting from alternations of magnetic flux in the core. (Their nature and the remedies to reduce these have already been discussed)

↑ is not dependent upon load current.

$P_c = P_h + P_e$

→ It may be emphasized there that the core-loss is constant for a Transformer operated at constant voltage and constant frequency as are all power frequency T/f

Copper-loss (I^2R-loss) → This loss occurs in winding resistances when the transformer carries the load current; varies as the square of the loading expressed as a ratio of the full-load.

↑ is depend upon the load current

$ratio = \frac{actual\ load}{full\ load}$

These are all ratio → $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{1}$ eg. $\frac{1}{2}$ full-load = Copper loss is $= (\frac{1}{2})^2$ of full load copper loss.

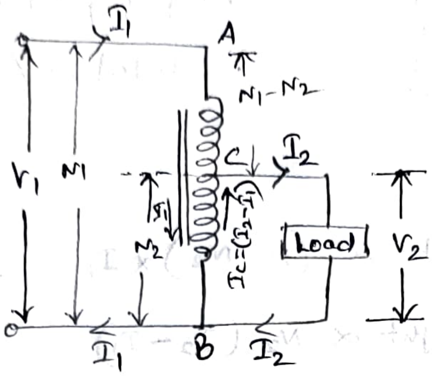
Load (stray)-loss → It largely results from leakage fields inducing eddy-current in the tank wall, and conductors.

Dielectric-loss → The seat (source) of this loss is in the insulating materials, particularly oil and solid insulations.

↑ rubber, pvc, Impregnated paper, porcelain

AUTOTRANSFORMER:

- It is a transformer with one winding only (is common to both the primary and secondary circuits), is called an auto transformer.
- The primary and secondary windings are electrically connected so that a part of the winding is common to the two, the transformer is known as an auto transformer.



note: AB is Py winding having N_1 turns and BC is secondary winding having N_2 turns.

- (But its theory and operation are similar to those of a two-winding transformer.)

- Such a T/f is practically economical where the voltage ratio is less than 2 in which case of electrical isolation of the two winding is not essential.

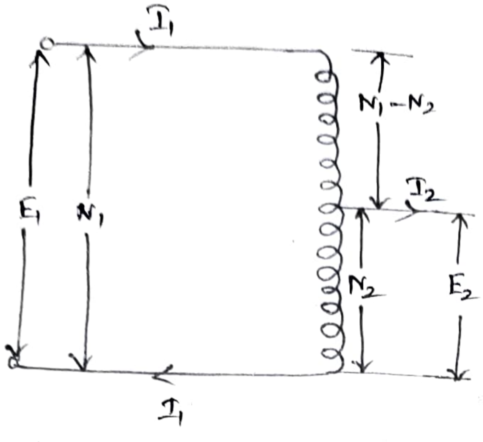
Applications

- It is used in induction motor starters,
- Inter connection of HV systems at voltage levels with ratio less than 2, and in obtaining variable voltage power supplies (low voltage and current levels)

- The auto Transformer has lower reactance, lower losses, smaller exciting current and better voltage regulation compared to its two winding counterpart.

PART-C

8 (a) prove that amount of copper saved in auto transformer is (1-K) times that of ordinary Transformer.



ordinary T/f

weight of Cu on py $\propto N_1 I_1$

weight of Cu on sy $\propto N_2 I_2$

Total $\propto (N_1 I_1 + N_2 I_2)$

Auto T/f

Top section Cu weight $\propto (N_1 - N_2) \times I_1$

The bottom section Cu, weight $\propto N_2 (I_2 - I_1)$

Total Cu in Auto T/f $\propto (N_1 - N_2) \times I_1 + N_2 (I_2 - I_1)$

$\propto I_1 N_1 - I_1 N_2 + I_2 N_2 - I_1 N_2 = I_1 N_1 - 2 I_1 N_2 + I_2 N_2$

$\propto (N_1 - 2N_2) I_1 + N_2 I_2$

$$S = \frac{\text{Auto T/f weight } (W_A)}{\text{ordinary T/f weight } (W_T)} = \frac{I_1 (N_1 - 2N_2) + N_2 I_2}{N_1 I_1 + N_2 I_2}$$

$$= \frac{(N_1 I_1 - 2 N_2 I_1) + I_2 N_2}{N_1 I_1 + N_2 I_2}$$

Divided by $N_2 I_1$

$$S = \frac{\left[\frac{N_1 \cancel{I_1}}{N_2 \cancel{I_1}} - \frac{2\cancel{N_2 I_1}}{\cancel{N_2 I_1}}\right] + \frac{I_2 \cancel{N_2}}{\cancel{N_2} I_1}}{\frac{N_1 \cancel{I_1}}{N_2 \cancel{I_1}} - \frac{\cancel{N_2} I_2}{\cancel{N_2} I_1}}$$

$=$

$$S = \frac{\left(\frac{N_1}{N_2} - 2\right) + \frac{I_2}{I_1}}{\frac{N_1}{N_2} + \frac{I_2}{I_1}}$$

Let $\frac{N_1}{N_2} = \frac{1}{K} = \frac{I_2}{I_1}$

$$\therefore \quad S = \frac{\left[\left(\frac{1}{K} - 2\right) + \frac{1}{K}\right]}{\frac{1}{K} + \frac{1}{K}} = \frac{1 - 2K + 1}{K} \Big/ \frac{2}{K} = \frac{\frac{2-2K}{K}}{2/K} = \frac{2-2K}{K} \times \frac{K}{2}$$

$$S = \frac{2 - 2K}{2} = \frac{\cancel{2}(1-K)}{\cancel{2}} = 1 - K$$

$$\boxed{S = 1 - K}$$

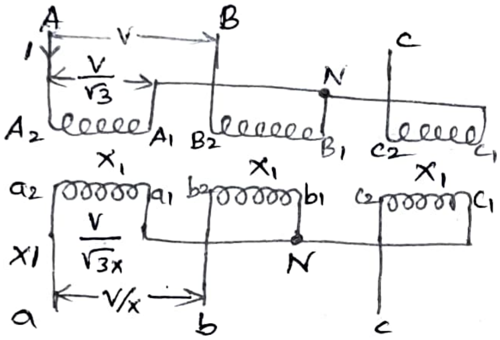
(or)

THREE PHASE Connections:-

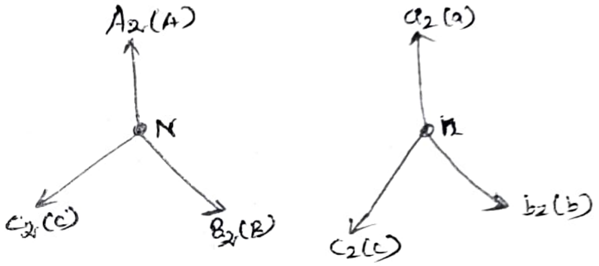
* A Verity q Connection's on each side (py, sy) q a 3ϕ T/f

* The 3ϕ (Threephases) can be Connected in star, Delta, open delta or zig zag star.

(i) Star/star (Y/Y) Connections.



phaser diagram

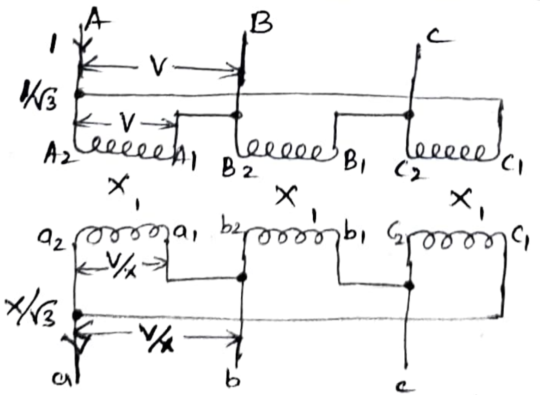


STAR/Star 0° connecting

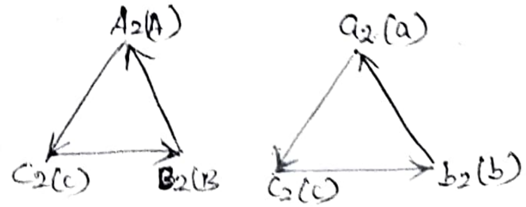
* The phase Transformation ratio is x:1
* The line transformation (line-to-line voltages, line currents) the ratio is also x:1.
* The voltages q the corresponding phases are in phase, this is known as 0°-Connections.
* If the winding terminals on sy side are reversed, the 180°- connections is obtained.

Advantages: (i) The presence q Neutral point so it is suitable for three phase and four wire systems.

(ii) Delta/Delta (Δ/Δ) connections.



phaser diagram.

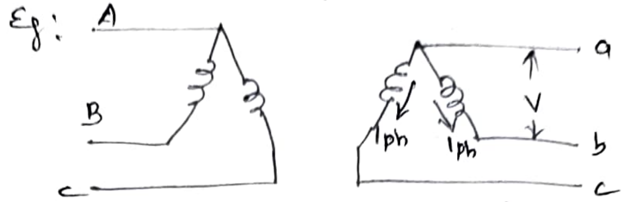


Delta/Delta Connections.

* The py and sy line voltages are in phase so it is the 0°- connection (is seen from phaser diagram)
* The sy voltages are in phase opposition to the primary voltages Com be visualized from the phasor diagram. This is the 180°- Connections.

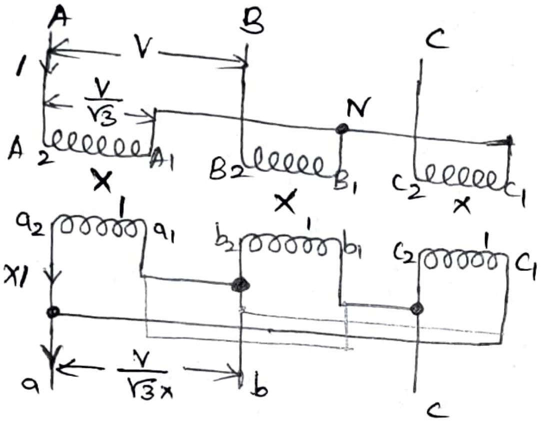
* The phase transformation ratio is $x:1$, the transforma ratio for line quantities is also $x:1$.

* In Delta/Delta connections if one of the Transformers is disconnected, the resulting connection is known as open-delta Eg:

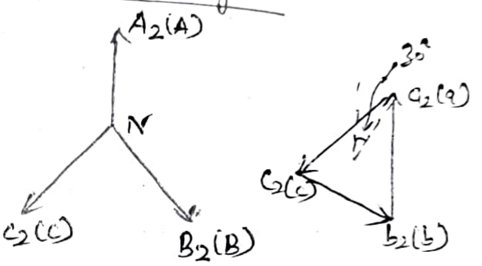


open-delta (or) V connection.

(iii) STAR/Delta (Y/Δ) connection.



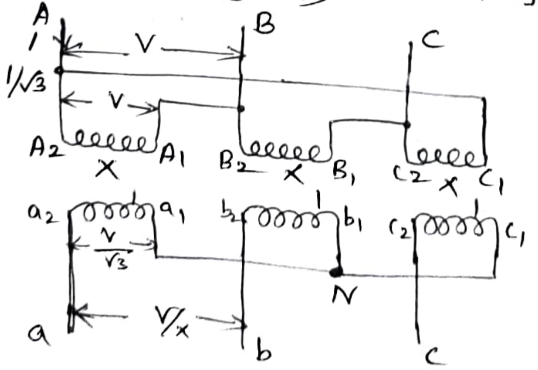
Phaser diagram.



Star/Delta - 30° connections

* The phase transformation ratio of the star/delta connection is $x:1$, the line transformation ratio in magnitude is $\sqrt{3} : x/1$

(iv) Delta/star (Δ/Y) connections.



* The phase transformation ratio is $x:1$, the transformation ratio is ~~x:1~~ of the line quantities will be $(x/\sqrt{3}):1$

SCOTT CONNECTION (PHASE CONVERSION)

2. A three –phase supply can be converted to a two-phase supply by connecting two single phase transformer of a suitable ratio is known as Scott connection.

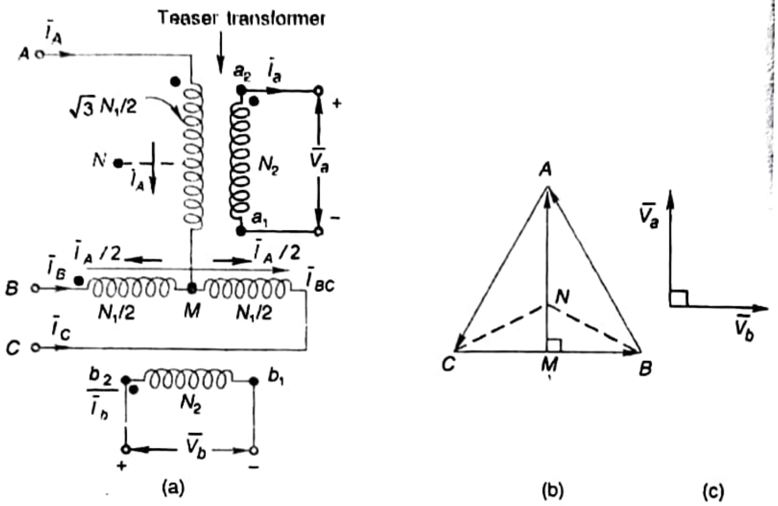


Fig. Scott connection

3. Phase conversion from three to two phases is needed in special case, such as in supplying 2-phase electric arc furnaces.

4. A 2-phase supply could thus be obtained by means of transformers; one connected across AM, called the teaser transformer and the other connected across the lines Band B C.

5. Since V_{AM}=($\sqrt{3/2}$) V_{BC} ,the transformer primary must have $\sqrt{3}$ N_1/2(teaser) and N_1 turns ;this would mean equal voltage /turns ratio in each transformer .

6. A balanced 2-phase supply could then be easily obtained by having both secondaries with equal number of turns, N2.

7. The point M is located midway on the primary of the transformer connected across the lines B and C.

8. The connection of two such transformers, known as Scott connection.